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MAT 222
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Quiz 3

"You know, it's funny. When you look at someone through rose-colored glasses, all the red flags just look like flags."
– Wanda Pierce, BoJack Horseman

Problem 1: Some researchers performed a hypothesis test of the claim that the mean SAT Mathematics score of California High School seniors was more than 490. Assume the population standard deviation was 100. They randomly selected 25 seniors from a California High School and decided to use a significance level of $\alpha = 0.01$.

(a) What is the probability of a Type I error?

(b) What is the power of this test to detect $\mu = 560$?

(c) What is the probability of a Type II error if $\mu = 560$?

(d) What theorem do you need for (a)–(c)? What are other possible error sources?

Solution.

(a) $P(\textit{Type I Error}) = \alpha = 0.01$.

(b) We have

$$\begin{cases} H_0 : \mu = 490 \\ H_a : \mu > 490 \end{cases}$$

We reject if the probability of $\overline{x}$ is 0.01 or less. This corresponds to $z = 2.33$. Then using $z = \frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$, we have

$$2.33 = \frac{\overline{x} - 490}{100/\sqrt{25}},$$

which means $\overline{x} = 536.6$. Now to calculate the power, we need to reject the null hypothesis (this happens when $\overline{x} > 536.6$). If $\mu = 560$, then we can calculate the probability as follows

$$z = \frac{536.6 - 560}{100/\sqrt{25}} = \frac{-23.4}{20} = -1.17 \rightsquigarrow 0.1210$$

So that the power is $1 - 0.1210 = 0.8790$.

(c) We know $P(\textit{Type II Error}) = 1 - \textit{Power} = 1 - 0.8790 = 0.1210$.

(d) You use the sampling distribution so you need the CLT. Other possible error sources are that the population standard deviation was simply assumed to be 100 and that the study only used a sample from one High School to draw conclusions.