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MAT 222
Spring 2019
Homework 1

"And I knew exactly what to do. But in a much more real sense, I had no idea what to do."
–Michael Scott, The Office

Problem 1: Watch "Scientific Studies: Last Week Tonight with John Oliver (HBO). What did you learn from the video? What did you like or dislike about the video? What does this say about Statistic's role in society?

Answers will vary

Problem 2: Watch The Mathematics of History. What did you learn from the video? What did you like or dislike about the video? What implications does the proposals in the video have for Statistics and its role in society? What is question you would like to see investigated using Statistics (or Mathematics generally) in typically non-quantitative fields?

Answers will vary

Problem 3: Each of the type of expressions below appeared in various statistical computations in MAT 221. Use a calculator to compute each of them.

(a) $\dfrac{110.4 - 98.4}{4.7} = \underline{\quad 2.55 \quad}$

(b) $\dfrac{1034 - 1000}{70/\sqrt{19}} = \underline{\quad 2.12 \quad}$

(c) $\left(\dfrac{1.96 \cdot 5.3}{0.8}\right)^2 = \underline{\quad 168.61 \quad}$

(d) $\dfrac{0.34 \cdot 0.54}{0.76 \cdot 0.89} = \underline{\quad 0.271 \quad}$

Problem 4: Each of the type of expressions below will appear in various statistical computations in MAT 222. Use a calculator to compute each of them.

(a) $\dfrac{5.6 - 7.2}{\sqrt{\dfrac{1.3^2}{12} + \dfrac{2.3^2}{14}}} = \underline{\quad -2.22 \quad}$

(b) $\dfrac{587 - 560}{78.4 \cdot \sqrt{\dfrac{1}{22} + \dfrac{1}{16}}} = \underline{\quad 1.05 \quad}$

(c) $\dfrac{(22-1) \cdot 78.6^2 + (16-1) \cdot 77.3^2}{22 + 16 - 2} = \underline{\quad 6093.51 \quad}$

(d) $\sqrt{\dfrac{0.65(1-0.65)}{21} + \dfrac{0.60(1-0.60)}{18}} = \underline{\quad 0.1554 \quad}$

Problem 5: Suppose you are examining a population which is normally distributed with mean $\mu = 409.7$ and standard deviation $\sigma = 26.4$, i.e. $N(409.7, 26.4)$. Based on this information, complete the following chart:

x	z	Probability
434	0.92	$P(X \leq x) = 0.8212$
363.5	-1.75	$P(X \leq x) = 0.0401$
411.02	0.05	$P(X \geq x) = 1 - 0.5199 = 0.4801$
465.4	2.11	$P(X \geq x) = 1 - 0.9826 = 0.0174$
388.844	-0.79	$P(X \geq x) = 1 - 0.2148 = 0.7852$

Problem 6: Suppose the score of a certain SAT were normally distributed with mean $\mu = 854$ and standard deviation $\sigma = 240$. What score (or lower) would put you in the bottom 30% of test takers? What was the minimal score one would need to get to be in the top 4% of test takers?

To be in the bottom 30%, you would need $P(X \leq x) = 0.30$*, this corresponds to a* z*-score of* $z = -1.88$*. But then we have*

$$\begin{aligned} -0.525 = z &= \frac{x - \mu}{\sigma} \\ -0.525 &= \frac{x - 854}{240} \\ -126 &= x - 854 \\ x &= 728 \end{aligned}$$

So to be in the bottom 30% of test takers, one would have to score 728 *or lower.*

To be in the top 4% of test takers, you would need $0.04 = P(X \geq x) = 1 - P(X \leq x)$ *so that* $P(X \leq x) = 0.96$*. This corresponds to a* z*-score of* 1.75*. But then we have*

$$\begin{aligned} 1.75 = z &= \frac{x - \mu}{\sigma} \\ 1.75 &= \frac{x - 854}{240} \\ 420 &= x - 854 \\ x &= 1274 \end{aligned}$$

So to be in the top 4% of test takers, you would need to score at least 1274*.*

Problem 7: If $x_1 = 2.1$, $x_2 = 3$, $x_3 = 4.1$, $x_4 = -1.1$, and $\overline{x} = 3$, find $\sum_{i=1}^{4} (x_i - \overline{x})^2$.

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
2.1	-0.9	0.81
3	0	0
4.1	1.1	1.21
-1.1	-4.1	16.81

Therefore, $\sum_{i=1}^{4} (x_i - \overline{x})^2 = 0.81 + 0 + 1.21 + 16.81 = 18.83.$