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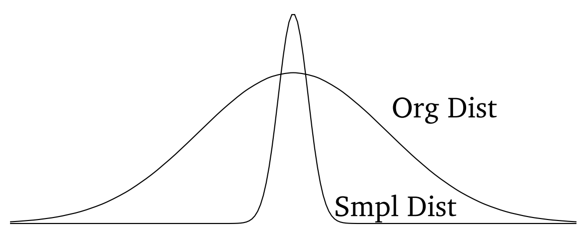
MAT 222
Spring 2019
Homework 2

"You mean I'm responsible for my own happiness? I can't even be responsible for my own breakfast"
–BoJack Horseman, BoJack Horseman

Problem 1: Explain what the Central Limit Theorem says. When does it apply?

The Central Limit Theorem (CLT) states that if you take a simple random sample (SRS) of size n, sufficiently large (usually $n \approx 30 - 40$ or larger), from any *population with mean μ and standard deviation σ, then the sampling distribution of group means, $\overline{x}$, is approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$, i.e. the sampling distribution is $N(\mu, \sigma/\sqrt{n})$.*

Problem 2: Suppose you take a random sample of 40 people from a population with $\mu = 80$ and $\sigma = 5$. On the same plot, sketch the population distribution and the sampling distribution on the same plot. Find the distribution of the sampling distribution. Justify your answer.



The CLT applies, so the sampling distribution has distribution $N(80, 5/\sqrt{40}) = N(80, 0.79)$.

Problem 3: What are the ways to decrease the margin of error of a confidence interval?

- *Increase the sample size, n*
- *Decrease the confidence level*
- *Decrease the standard deviation, σ*

Problem 4: What is problematic about this statement, “There is a 95% chance that the population mean is between 24.7 and 25.4”?

The statement makes it seem like the population mean can vary, when there is one true *fixed mean out there. This is why “we are 95% certain that the population mean is between 24.7 and 25.4” is ‘better’.*

Problem 5: Suppose you are examining a population which is normally distributed with mean $\mu = 87.1$ and standard deviation $\sigma = 6.9$, i.e. $N(87.1, 6.9)$. Suppose you take a sample of size 57 from this distribution and look at the sample group mean $\overline{x}$. Based on this information, complete the following chart:

$\overline{x}$	z	Probability
87.94	0.92	$P(X \leq x) = 0.8212$
85.5	-1.75	$P(X \leq x) = 0.0401$
87.15	0.05	$P(X \geq x) = 1 - 0.4801 = 0.5199$

Problem 6: Tropical swam-founding wasps rely on female workers to raise their offspring. One possible explanation for this strange behavior is inbreeding, which increases relatedness among the wasps, presumably making it easier for the workers to pick out their closest relatives as propagators of their own genetic material. To test this theory, 197 swam-founding wasps were captured in Venezuela, frozen at -70° C, and then subjugated to various tests (*Science*, Nov. 1988). The data were used to generate an inbreeding coefficient, c.f. Wright’s equation, with the following results: $\overline{x} = 0.044$ and $\sigma = 0.884$.

(a) Construct a 99% confidence interval for the mean inbreeding coefficient of this species of wasp. Justify your answer.

(b) A coefficient of 0 implies that the wasp has no tendency to inbreed. Perform a hypothesis test with significance level $\alpha = 0.01$ to determine if this coefficient is nonzero.

(c) How could you answer (b) using the confidence interval in (a)?

(a) $\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 0.044 \pm 2.576 \cdot \frac{0.884}{\sqrt{197}} = 0.044 \pm 0.162$. *Therefore, we are 99% certain that the true inbreeding coefficient is between* -0.118 *and* 0.206.

McClave, J.T. & Sincich, T. (2000) *Statistics* (8th Ed). Upper Saddle River, NJ: Prentice-Hall

(b) $\begin{cases} H_0 : \mu = 0 \text{ (no inbreeding)} \\ H_a : \mu \neq 0 \text{ (some inbreeding)} \end{cases}$

$$z = \frac{0.044 - 0}{0.884/\sqrt{197}} = 0.70 \rightsquigarrow 0.7580$$

Then $P(\overline{x} \neq 0) = 2 \cdot P(\overline{x} \geq 0) = 2 \cdot (1 - 0.7580) = 0.4840 \not\leq 0.01$. *Therefore, there is not sufficient evidence to reject the null hypothesis, i.e. there is not sufficient evidence to suggest that this species of wasp inbreeds.*

(c) *Yes, we are 99% certain that the true inbreeding coefficient is between* -0.118 *and* 0.206*, and this interval contains 0. So an inbreeding coefficient of 0 is statistically possible.*