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MAT 222
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Homework 3

"Blackmail is such an ugly word. I prefer extortion. The 'x' makes it sound cool."
–Bender, Futurama

Problem: A education policy maker is interested in how NYS High School students performed on the SAT Mathematics portion. She decides to take a SRS of 400 students. In order to see if the mean SAT Mathematics score has increased from the mean score of 502 in 2009, she tests

$$\begin{cases} H_0 : \mu = 502 \\ H_a : \mu > 502. \end{cases}$$

Assuming the population standard deviation is $\sigma = 100$ (based on information from the SAT Board), she decides to use a significance level of 0.01 that rejects H_0 if $\frac{\overline{x}-502}{100/\sqrt{400}} \geq 2.326$, or equivalently if $\overline{x} \geq 513.63$.

(a) Find $P(\text{Type I error})$.

(b) Find the probability of failing to reject the null hypothesis, if the null hypothesis is consistent with the data.

(c) Find the probability of a Type II error if $\mu = 512$.

(d) Is this test sufficiently sensitive to detect an increase of 10 points in the population mean of the SAT Mathematics score?

Solution.

(a) $P(\textit{Type I error}) = \alpha = 0.01$.

(b) $P(\textit{fail to reject} \mid H_0 \textit{ true}) = 1 - P(\textit{Type I error}) = 1 - \alpha = 0.99$.

(c) Recall, $P(\textit{Type II error}) = P(\textit{fail to reject} \mid \mu = 512)$. *Given* $\alpha = 0.01$*, we reject the null hypothesis whenever* $\overline{x} \geq 513.63$*. So we fail to reject when* $\overline{x} \leq 513.63$*. Then* $P(\textit{Type II error})$ *happens with probability*

$$\frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{513.63 - 512}{100/\sqrt{400}} = \frac{1.63}{5} = \frac{1.63}{5} = 0.326 \rightsquigarrow \frac{0.6255 + 0.6293}{2} = 0.6274,$$

i.e. the probability of a Type II error is 0.6274.

(d) Recall the power is the probability of rejecting the null hypothesis when (in this case) $\mu = 512$*; that is, the power is the ability to detect when the true mean is (in this case) 512. We know power* $= 1 - P(\textit{Type II error}) = 1 - 0.6274 = 37.26\%$*, i.e. there is only a 37.26% chance that the test will 'notice' if the true mean is 512. So this test is probability not sufficiently sensitive to detect an increase of 10 points in the mean (causing it to rise from 502 to 512).*