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MAT 222
Spring 2019
Homework 5

"Sticks and stones may break your bones, but words leave psychological wounds that will never heal."
–Mr. Turner, Fairly Odd Parents

Problem 1: Researchers are trying to determine if students learn better from print textbooks or reading material from a digital screen. They test students two at a time, choosing equal ability students, giving them a short article to read (one student reading a printed version and the other reading a digital version), and then giving each identical exams to test their recall. The differences in the scores (print − digital) were

$$2,\ 3,\ -1,\ 0,\ -4,\ -1,\ 1,\ 1,\ 4,\ -3,\ 0,\ -1,\ 2,\ 0,\ 3,\ 3,\ -4,\ -2,\ 2.$$

(a) What type of statistical test do you need to apply in this case? Justify your answer completely.

(b) Find a 99% confidence interval for the difference of the scores. Summarize your results in a sentence.

(c) State and test an appropriate hypothesis test using $\alpha = 0.05$. Be sure to state your test statistic, p-value, and conclusion.

Solution.

(a) We need to use a matched pairs t-test. This is because the groups were not chosen independently — students were chosen by pairing equal ability students together. Therefore, any ordinary two-sample t-test would not be appropriate.

(b) Examining the differences, we find $n = 19$, $\overline{x} = 0.26$, and $s = 2.4$. Therefore, we have dof$=$ $19 - 1 = 18$ so that $t^* = 2.878$. Then we have

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}} = 0.26 \pm 2.878\, \frac{2.4}{\sqrt{17}} = 0.26 \pm 2.878(0.551) = 0.26 \pm 1.6,$$

so that we are 99% certain that the true difference between the average print reader score and average digital reader score is between -1.34 and 1.86, i.e. we are 99% certain that on average print readers score between -1.44 points worse to 1.96 points better than the average digital reader.

(c) We test

$$\begin{cases} H_0 : \mu_{diff} = 0 \\ H_a : \mu_{diff} \neq 0 \end{cases}$$

We have

$$t = \frac{0.26 - 0}{2.4/\sqrt{17}} = \frac{0.26}{0.551} = 0.471 \overset{dof\ 18}{\rightsquigarrow} 0.25,$$

so that $p = 2 \cdot 0.25 = 0.50$. Since $p \not< \alpha$, we fail to reject the null hypothesis. Therefore, the data is consistent with the fact that both test takers score equally well on average.

Problem 2: Suppose you are testing

$$\begin{cases} H_0 : \mu = 110 \\ H_a : \mu < 110 \end{cases}$$

with significance level $\alpha = 0.10$. If $n = 28$, $\overline{x} = 99$, and $s = 7$, find

(a) The probability of a Type I error.

(b) The probability of a Type II error if $\mu = 105$.

(c) The power of this test if $\mu = 105$.

Solution.

(a) $P(\textit{Type I Error}) = 0.10$.

(b) We reject if the probability of $\overline{x} < 110$ *is* 0.10 *or less. We have dof*$= 28 - 1 = 27$*. Then we need* $t = -1.314$*. Therefore, we have* $-1.314 = \frac{\overline{x}-110}{7/\sqrt{28}}$ *so that* $\overline{x} = 108.262$*. For a Type II error, we need to fail to reject. So we need* $\mu = 105$ *and* $\overline{x} > 108.262$*. We have*

$$t = \frac{108.262 - 105}{7/\sqrt{28}} = \frac{3.262}{1.323} = 2.466 \overset{\textit{dof } 27}{\rightsquigarrow} = 0.02.$$

Therefore, $P(\textit{Type II Error}) = 0.02$.

(c) We know Power$= 1 - P(\textit{Type II Error}) = 1 - 0.02 = 0.98$.

Problem 3: To study the effectiveness of their weight loss programs, a local dieting center chooses 24 subjects who were at least 20% overweight to take part in a three month diet support program. The subjects were divided into three different programs: the first group received no support, the second group received phone support, and the third received in-person support. Private weightings determined each subjects weight at the beginning of the program and four months after the program's end. The data of the total weight loss from these three groups at the end of the experiment is summarized below.

Program	$\overline{x}$	s
1	11.3	4.33
2	7.2	11.201
3	14.1	4.87

(a) Would a pooled t-test be appropriate to compare program one with program two? What about program two with program three? Justify your response.

(b) Why might it be appropriate to use a pooled t-test to compared program one with program three? Justify your response.

(c) Use a pooled t-test to construct a 96% confidence interval for the difference in the mean for program one and program three.

(d) Use a pooled t-test to test the hypothesis that there is no difference between diet support and in-person diet support with a significance level of $\alpha = 0.3$. State your test statistic, p-value, and conclusion.

Solution.

(a) Because $4.33/11.201 = 0.387$ and $11.201/4.87 = 2.3$, neither being between 0.5 and 2, it is inappropriate to use a pooled t-test.

(b) Observe that $0.5 < 4.33/4.87 = 0.889 < 2$, so that a pooled t-test might be appropriate.

(c) We use difference program three − program one. Note that $n_1 = n_2 = 8$, so we have pooled standard deviation

$$s_p = \sqrt{\frac{(8-1)4.87^2 + (8-1)4.33^2}{8+8-2}} = \sqrt{\frac{297.2606}{14}} = 4.608.$$

We have dof$= n_1 + n_2 - 2 = 8 + 8 - 2 = 14$ so that $t^* = 2.264$. Therefore,

$$(\overline{x}_1 - \overline{x}_2) \pm t^* s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (14.1 - 11.3) \pm 2.264(4.608)\sqrt{\frac{1}{8} + \frac{1}{8}} = 2.8 \pm (10.4325)0.5,$$

so that we are 96% certain that the true difference between the average weight loss in program three and average weight loss in program one is between -2.42 lb and 8.02 lb.

(d) We have

$$\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 \neq \mu_2 \end{cases}$$

Then we have test statistic

$$t = \frac{(14.1 - 11.3) - 0}{4.608} = \frac{2.8}{4.608} = 0.608 \overset{\text{dof } 14}{\rightsquigarrow} 0.25.$$

Therefore, $p = 2 \cdot 0.25 = 0.50$*. Because* $p \not< \alpha$*, we fail to reject the null hypothesis; that is, the data is consistent that, on average, individuals in program one and program three lose the same amount of weight.*