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MAT 222
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Homework 6

"Cindy, the TV is leaking!"
–Brenda, Scary Movie 3

Problem 1: A software developer is interested in analyzing the proportion of CPAs who use a certain accounting software.

(a) How many observations should be taken to estimate, at a 95% confidence level, the population proportion within a margin of error of 0.03, if the assumed percentage is 0.34? How many should be used if no prior information about the proportion is available?

We have $m = 0.03$*,* $z^* = 1.96$*, and* $p^* = 0.34$*. Then we have*

$$n = \left(\frac{1.96}{0.03}\right)^2 0.34(1-0.34) = 957.839$$

so that 958 *people should be used. If no prior information is known, we have*

$$n = \frac{1}{4}\left(\frac{1.96}{0.03}\right)^2 = 1067.11$$

so that $1,068$ *people should be used.*

(b) A random sample of 200 CPAs was collected and eighty nine out of 200 in the sample reported using the specific software. Find a 95% confidence interval for the true proportion of CPAs who use that accounting software.

We have $\hat{p} = X/n = 89/200 = 0.445$*. Furthermore, we have* $z^* = 1.96$*. Then we have*

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.445 \pm 1.96\sqrt{\frac{0.445(1-0.445)}{200}} = 0.445 \pm 0.0689$$

Therefore, we are 95% certain that the true proportion of CPAs that use the software is between 0.376 *and* 0.514*.*

Problem 2: Arthritis is a painful, chromic inflammation of the joints. An experiment on the side effects of pain relievers examined arthritis patients to find the proportion of patients who suffer side effects when using ibuprofen to relieve the pain. If more than 3% of users suffer side effects, the Food and Drug Administration will put a stronger warning label on packages of ibuprofen. The experiment recruited 440 subjects with chronic arthritis who were given ibuprofen for pain relief, with 23 subjects suffering from adverse side effects. Does this data provide statistically significant evidence at the 0.01 level that less than 3% of those who use ibuprofen for arthritis suffer adverse side effects? Note your hypotheses, test statistic, p-value and/or critical value, and state your conclusions in the context of the problem.

Solution. *Let p denote the proportion of users with chronic arthritis that will experience adverse side effects from regular ibuprofen use. Then we test*

$$\begin{cases} H_0 : p = 0.03 \\ H_a : p > 0.03 \end{cases}$$

We have $\hat{p} = X/n = 23/440 = 0.05227$. Then we have test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p\,(1-p)}{n}}} = \frac{0.05227 - 0.03}{\sqrt{\frac{0.03(1-0.03)}{440}}} = \frac{0.02227}{0.00813} = 2.74 \rightsquigarrow 0.9969.$$

Therefore, we have p-value $1 - 0.9969 = 0.0031$. Since $p \not< 0.01$, we fail to reject the null hypothesis. Therefore, the Food and Drug Administration should not place stronger warning labels on ibuprofen.

Problem 3: An internet sales site is interested in determining whether there is a difference between male and female users in terms of the respective proportions that use debit cards to buy products. A random sample of their users was collected which had 226 females and 192 males. 131 of the females and 89 of the males bought items using a debit card.

(a) Construct a 90% confidence of the difference of the proportion of men and women who use debit cards to make purchases.

We have $\hat{p}_M = 89/192 = 0.463542$ *and* $\hat{p}_F = 131/226 = 0.579646$. *We take difference* $D = \hat{p}_F - \hat{p}_M = 0.116104$. *We have standard error of the difference*

$$SE_D = \sqrt{\frac{0.579646(1-0.579646)}{226} + \frac{0.463542(1-0.463542)}{192}} = 0.0487164.$$

Since $z^* = 1.645$, *we have*

$$(\hat{p}_F - \hat{p}_M) \pm z^* SE_D = 0.116104 \pm 1.96 \cdot 0.0487164$$

so that we are 90% certain that females on average use debit cards for purchases between 2.06% and 21.16% more than males.

(b) State the appropriate null and alternative hypotheses to test the claim that there is a gender difference in terms of the respective proportions that buy the products using a debit card. Test this hypothesis at a significance level of 0.01, being sure to state the test statistic and p-value. State your result in the context of the problem.

The null and alternative hypotheses are

$$\begin{cases} H_0 : \hat{p}_F - \hat{p}_M = 0 \\ H_a : \hat{p}_F - \hat{p}_M \neq 0 \end{cases}$$

or equivalently

$$\begin{cases} H_0 : \hat{p}_F = \hat{p}_M \\ H_a : \hat{p}_F \neq \hat{p}_M \end{cases}$$

We have test statistic

$$z = \frac{\hat{p}_F - \hat{p}_M}{SE_D} = \frac{0.116104}{0.0487164} = 2.38 \rightsquigarrow 0.9913$$

and $1 - 0.9913 = 0.0087$. *Therefore, our* p*-value is* $p = 0.0174$. *Since* $p \nless 0.01$, *we fail to reject the null hypothesis. Therefore, there is no significant difference between the average percentage of males and females that use debit cards for their purchases.*