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MAT 222
Spring 2019
Homework 8

"My name is Ozymandias, King of Kings; Look on my Works, ye Mighty, and despair!"
–Percy Shelley, Ozymandias

Problem 1: The Mathematics Department is trying to predict students take-home exam scores from their in-home exam scores. They perform a test on 9 students and record their in-class and take-home exams scores to create a linear model. A few of the model statistics are reported below.

$$\begin{array}{lll} \overline{x} = 34.00 & s_x = 14.30 & \sum(x_i - \overline{x})^2 = 1636.0 \\ \overline{y} = 20.67 & s_y = 7.25 & \sum(y_i - \overline{y})^2 = 420.0 \\ r = 0.849 & s = 4.089 & \\ b_0 = 6.04 & b_1 = 0.43 & \end{array}$$

(a) Use s_x to confirm the value $\sum(x_i - \overline{x})^2 = 1636.0$.

$$\begin{aligned} s_x^2 &= \frac{1}{n-1}\sum(x_i - \overline{x})^2 \\ 14.30^2 &= \frac{1}{9-1}\sum(x_i - \overline{x})^2 \\ 14.30^2 \cdot 8 &= \sum(x_i - \overline{x})^2 \\ \sum(x_i - \overline{x})^2 &= 1635.92 \end{aligned}$$

(b) What was the resulting linear model for their statistical analyses?

$$\texttt{TAKE-HOME} = 6.04 + 0.43\ \texttt{IN-CLASS}$$

(c) What is the standard error for b_0?

$$SE_{b_0} = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{\sum(x_i - \overline{x})^2}} = 4.089\sqrt{\frac{1}{9} + \frac{34^2}{1636}} = 3.70$$

(d) Create a 99% confidence interval for the coefficient b_1.

We have $n = 9$ so that the degrees of freedom are $n - 2 = 7$ (this gives $t^* = 3.499$). We also have standard error

$$SE_{b_1} = \frac{s}{\sqrt{\sum(x_i - \overline{x})^2}} = \frac{4.089}{\sqrt{1636}} = 0.10$$

Then we compute $b_1 \pm t^* SE_{b_1} = 0.43 \pm 3.499(0.10)$ to find confidence interval $(0.08, 0.78)$.

(e) What is the value of SST?

We know SST= $\sum(y_i - \overline{y})^2$ *so that we have SST=* 420.0.

(f) What is the value of the coefficient of determination for this model? What does it tell you?

The coefficient of determination is $r^2 = 0.849^2 = 0.72$. *This gives the 'percent linearity' in the data, i.e. the percent of the variability in the data explained by the model.*

(g) What is the value of the MSE for this model?

We have MSE= $s^2 = 4.089^2 = 16.7199$.

(h) Create a 90% confidence interval for the take-home exam score of a student who receives an in-class exam score of 40.

Using the model, we have predicted take-home exam score of $\hat{y} = 6.04 + 0.43(40) = 23.24$. *Because* $n = 9$, *we have degrees of freedom* $n - 2 = 7$, *giving* $t^* = 1.895$. *We have standard error*

$$SE_{\hat{y}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \overline{x})^2}{\sum(x_i - \overline{x})^2}} = 4.089\sqrt{1 + \frac{1}{9} + \frac{(40 - 34)^2}{1636}} = 4.35$$

Computing $\hat{y} \pm t^* SE_{\hat{y}} = 23.24 \pm 1.895(4.35)$ *gives confidence interval* $(15.00, 31.48)$.

(i) What is the *mean* take-home exam score for a student who receives an in-class exam score of 40?

This is precisely what the model predicts, $\hat{\mu} = 6.04 + 0.43(40) = 23.24$.