Math 121: Exam 2
Summer – 2019
06/13/2019
100 Minutes

Name: *Caleb M^{c}Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 9 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	15	
4	15	
5	15	
6	10	
7	10	
8	15	
Total:	100	

1. (10 points) A 'werd' is any collection of letters in a row. For example, 'bird' is a werd and 'gzzyae' is a werd.

   (a) How many werds can be formed using the letters from the word 'smol'?

   $$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

   (b) How many werds can be formed using the letters from the word 'wisenheimer'.

   $$\frac{11!}{2!3!} = 3,326,400$$

   (c) How many werds can be formed using the letters from the word 'yeeting' if there must be exactly two letters between the 'y' and 'g'?

   Count the number of ways of arranging the letters that are not y and g ***and*** *the number of ways of placing the y and g* ***and*** *the number of ways of arranging the y and g in order. Then we must have...*

   $$\frac{5!}{2!} \cdot 4 \cdot 2 = 60 \cdot 4 \cdot 2 = 480$$

2. (10 points) A dice game at a fair lets you roll 5 die. If you roll three of a kind, you win a small prize; if you roll four of a kind, you win a medium prize; if you roll five of a kind, you win a large prize. Suppose the small prize is \$1, the medium prize is \$5, and the large prize is \$20. You only get to roll the dice once, and the chance to do so costs \$1. You have a friend that is clever at Math whom you call on the phone. They have a brief moment and manage to calculate a few of the probabilities for you, as shown in the table below. Based on all this information, answer the following questions.

Amount	\$0	\$1	\$5	\$20
Probability	0.787	0.1929	0.0193	0.0008

(a) Fill in the missing probability in the table above that your friend did not calculate before hanging up.

$$1 - 0.1929 - 0.0193 - 0.0008 = 0.787$$

(b) Calculate how much you would win/lose on average if you played the game 'many' times (rounding to the nearest cent). Based on this information, should you play this game? Explain.

$$\mu = \sum xP(x) = 0(0.787) + 1(0.1929) + 5(0.0193) + 20(0.0008) = 0.3054 \approx 0.31$$

No, you should not play this game. You only win on average $\$0.31$ *per game but you are paying* $\$1$ *per game. Therefore, on average, you are losing* $\$0.69$ *per game by playing.*

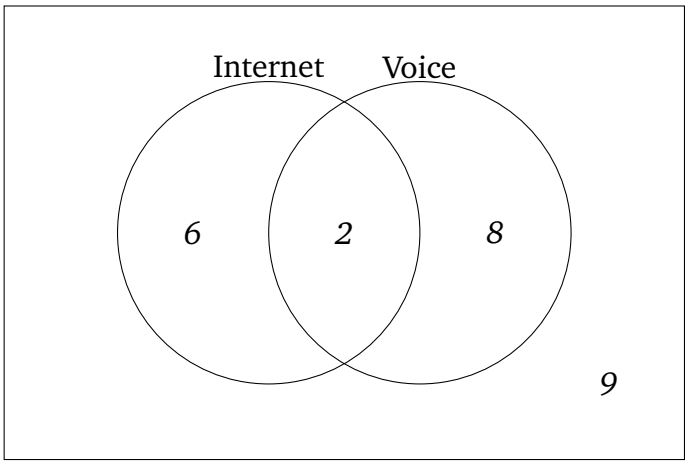
(c) Calculate the standard deviation (to the nearest cent) for the amount you would win on average if you played the game many times.

x	$x - \mu$	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
0	-0.31	0.0961	0.0756
1	0.69	0.4761	0.0918
5	4.69	21.9961	0.4245
20	19.69	387.696	0.3102
			Total: 0.9021

Therefore, $\sigma^2 = 0.9021$ *so that* $\sigma = \sqrt{0.9021} = 0.95$.

3. (15 points) WorstBuys offers 25 types of televisions. There are two special types of televisions they sell: 8 types of televisions which connect to the internet and 10 types of televisions that take voice commands. There are 2 types of televisions which can do both. Based on this information, answer the following questions:

   (a) Complete the following Venn diagram by filling in the appropriate numbers:



   (b) What is the probability that a randomly chosen TV can neither connect to the internet nor take voice commands?

$$\frac{9}{25} = 0.36$$

   (c) What is the probability that a randomly chosen TV can connect to the internet but cannot take voice commands?

$$\frac{6}{25} = 0.24$$

   (d) What is the probability that a randomly chosen TV which can take voice commands is able to connect to the internet?

$$\frac{2}{2+8} = \frac{2}{10} = 0.20$$

4. (15 points) You take a survey of men at a college, asking how they would describe their friends using adjectives from a preselected list. The results (broken down by major) are shown below.

	'Boi'	'Bro'	'Dude'	'Dudebro'	Total
STEM	1	7	2	6	16
Humanities & Arts	5	6	5	7	23
Social Sciences	7	15	1	2	25
Total	13	28	8	15	64

(a) What is the probability that a randomly chosen person from this survey was in the Social Sciences?

$$\frac{25}{64} = 0.3906$$

(b) What is the probability that a randomly chosen person from this survey was in STEM and used the phrase 'Dudebro'?

$$\frac{6}{64} = 0.0938$$

(c) What is the probability that a randomly chosen person from this survey used the term 'Boi' or was in the Humanities and Arts?

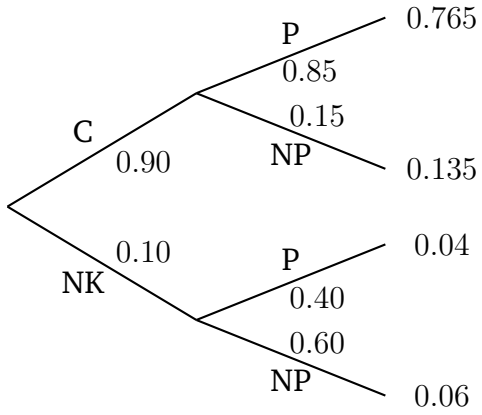
$$\frac{1+5+7+6+5+7}{64} = \frac{13+23-5}{64} = \frac{31}{64} = 0.4844$$

(d) If a person from this survey used the term 'Bro', what is the probability that they came from the Social Sciences?

$$\frac{15}{7+6+15} = \frac{15}{28} = 0.5357$$

5. (15 points) At Cornell University, everyone has watched The Office. [They're cool like that.] Of Cornell students, 90% of students know that Andy Bernard went to Cornell. Of the students that know he went to Cornell, 85% know his catchphrase—"rudit-dit-dodoo." Even from students that do not know that Andy went to Cornell, 40% will still know Andy's catch phrase.

   (a) Based on the information above, fill in the following tree diagram completely with the correct probabilities . [Note: C stands for 'knows Andy went to Cornell', NK stands for 'does not know Andy went to Cornell', P stands for 'knows Andy's catchphrase', and NP stands for 'does not know Andy's catchphrase.']



   (b) What is the probability that a randomly chosen Cornell student does not know Andy went to Cornell?

$$0.04 + 0.06 = 0.10$$

   (c) What is the probability that a randomly chosen Cornell student knows Andy's catchphrase?

$$0.765 + 0.04 = 0.805$$

   (d) What is the probability that a randomly chosen Cornell student knows that Andy went to Cornell or knows his catchphrase?

$$0.765 + 0.135 + 0.04 = 0.94$$

   (e) If a Cornell student knows Andy's catchphrase, what is the probability that they know he went to Cornell?

$$\frac{0.765}{0.765 + 0.04} = \frac{0.765}{0.805} = 0.95$$

6. (10 points) You are in a room filled with 10 people.

   (a) How many committees consisting of 6 people can be formed from these people?

$$\binom{10}{6} = {}_{10}C_6 = 210$$

   (b) How many ways can you choose people from this group for awards 'Best Dressed', 'Most Likely to Succeed', and 'Most Likely to Pass MAT 121'? [A person may win more than one award.]

$$10^3 = 1,000$$

   (c) How many ways can you choose two distinct people from this group to serve as an instructor and assistant for a class?

$${}_{10}P_2 = 10 \cdot 9 = 90$$

   (d) How many committees of 7 people with a designated president, vice president, and secretary can be formed, assuming no person can hold more than one of these special positions?

$${}_{10}C_7 \cdot {}_7P_3 = 120 \cdot 210 = 25,200$$

7. (10 points) At Syracuse University, 80% of students own a White Geese jacket. You ask 18 students at random whether they own one of these jackets.

   (a) What is the probability that *exactly* 16 of the students own a White Geese jacket?

$$ {}_{18}C_{16}\,(0.80)^{16}\,(0.20)^{2} = 153 \cdot 0.0281475 \cdot 0.04 = 0.1723 $$

   (b) What is the probability that more than 16 of these students own a White Geese jacket?

$$ {}_{18}C_{17}\,(0.80)^{17}\,(0.20)^{1} + {}_{18}C_{18}\,(0.80)^{18}\,(0.20)^{0} = 0.0811 + 0.0180 = 0.0991 $$

   (c) What is the probability that at most 16 of these students own a White Geese jacket?

$$ P(\textit{at most 16}) = 1 - P(\textit{more than 16}) = 1 - 0.0991 = 0.9009 $$

8. (15 points) Based on the information about events A, B, C below, answer the following questions:

$$P(A) = 0.40 \qquad P(B) = 0.80 \qquad P(C) = 0.20 \qquad P(A \cap C) = 0.10$$

(a) Are A and B disjoint? Explain.

No. If they were disjoint, then $P(A \cup B) = P(A) + P(B) = 0.4 + 0.8 = 1.2$, which is impossible.

(b) Are A and C disjoint? Explain.

No. $P(A \cap C) = 0.10$ so that they must share events in common.

(c) Are A and C independent? Explain.

No. If they were independent, then $P(A \cap C) = P(A)P(C)$. But $P(A \cap C) = 0.10$ and $P(A)P(C) = 0.4 \cdot 0.2 = 0.08$.

(d) Find the value of $P(\overline{C})$.

$P(\overline{C}) = 1 - P(C) = 1 - 0.20 = 0.80$.

(e) Find the value of $P(A \cup C)$.

$P(A \cup C) = P(A) + P(C) - P(A \cap C) = 0.40 + 0.20 - 0.10 = 0.50$.

(f) Assuming events A, B are independent, find the value of $P(A \cap B)$.

If A, B are independent, then $P(A \cap B) = P(A)P(B) = 0.40 \cdot 0.80 = 0.32$.

(g) Find the value of $P(C \mid A)$.

$$P(C \mid A) = \frac{P(A \cap C)}{P(A)} = \frac{0.10}{0.40} = 0.25.$$

(h) If instead of $P(A \cap C) = 0.10$, we had $P(A \cap C) = 0$. Could A and C be independent? Explain.

No. If $P(A \cap C) = 0$, then A and C are disjoint. But disjoint events are never independent.