Math 121: Final Exam
Summer – 2019
06/27/2019
145 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 13 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	25	
7	25	
8	20	
9	20	
10	10	
11	20	
12	15	
13	25	
Total:	185	

1. (5 points) Draw a stem-and-leaf plot for the following dataset:

$$7.8,\quad 9.7,\quad 8.4,\quad 4.7,\quad 7.5,\quad 0.2,\quad 6.0,\quad 5.0,\quad 4.1,\quad 8.5,\quad 6.1,$$
$$8.6,\quad 7.9,\quad 6.0,\quad 5.5,\quad 3.0,\quad 8.5,\quad 9.9,\quad 7.3,\quad 3.1,\quad 2.4,\quad 9.3$$

Using the stem-and-leaf plot, indicate whether the data is symmetric, left skewed, or right skewed.

0	2
1	
2	4
3	0 1
4	1 7
5	0 5
6	0 0 1
7	3 5 8 9
8	4 5 5 6
9	3 7 9

Stem Unit: 1

The dataset is clearly left skewed.

2. (5 points) In the blanks provided, indicate whether the underlined measurement represents a quantitative or categorical variable.

   (a) *Categorical* : That's so ratchet.

   (b) *Categorical* : My favorite type of clothing is anything flannel.

   (c) *Quantitative* : That is their sixth grande, iced, sugar-free, vanilla latte with soy milk today...

   (d) *Categorical* : The researcher entered males and females into the spreadsheet as 0's and 1's, respectively.

   (e) *Quantitative* : Kayne's $120 plain white t-shirts are not selling that well...

3. (5 points) In the blanks provided, indicate whether the underlined measurement represents a discrete or continuous variable.

   (a) *Continuous* : I am 79 inches tall.

   (b) *Discrete* : There are twelve The Fast and the Furious movies, but no good ones.

   (c) *Continuous* : My professor told me probabilities are between 0 and 1.

   (d) *Discrete* : The digital clock on the wall reads 4:20 pm.

   (e) *Continuous* : There are 925,600 minutes in a year. How do you measure, measure a year?

4. (5 points) Indicate in the spaces provided whether the described experiment represents a convenience, systematic, stratified, cluster, or random sample.

   (a) ___*Convenience*___ : A newspaper polled its readers on their political views.

   (b) ___*Random*___ : Your coming was foretold by gyromancy.[1]

   (c) ___*Systematic*___ : Every other person from the class roster was emailed.

   (d) ___*Cluster*___ : A research firm interviewed all the HS teachers from 3 randomly chosen High Schools to gather information about county classroom needs.

   (e) ___*Stratified*___ : People in a crowd were divided based on music preferences, then a few people from each preference were interviewed.

5. (5 points) In the blanks provided, indicate whether the underlined measurement represents a nominal, ordinal, interval, or ratio level measurement.

   (a) ___*Nominal*___ : My name is <u>Inigo Montoya</u>. You killed my father. Prepare to die.

   (b) ___*Interval*___ : Oscar Martinez comes in early to set the thermostat to <u>66°F</u>. Some people may not like it so cool. But he doesn't care.

   (c) ___*Ordinal*___ : *Finding Nemo* was the <u>third</u> best Pixar movie.

   (d) ___*Ratio*___ : "I am just one stomach flu away from my goal <u>weight</u>." —Emily Charlton

   (e) ___*Nominal*___ : "It's not a <u>man</u> purse. It's called a satchel. Indiana Jones wears one." —Alan Garner

[1] Gyromany is where one stands in a center of a circle with letters around the edge. You spin until you are so dizzy you fall down. The letter fallen on is the letter chosen. You continue this until a message is spelled out.

6. (25 points) A researcher is studying the affects of Monopoly® on friendship. The researcher records the number of minutes until someone flips the board. The dataset is given below.

1	6 7
2	5 6
3	0 6 9
4	1 1 5 8
5	0 1 3 4 5 5 6
6	4 4 6
7	1 1 4
8	0 1

Stem Unit: 10

(a) What is P_{35} for this dataset?

$$P_{35} = \frac{35}{100} \cdot 26 = 9.1 \rightsquigarrow 10\textit{th value} = 45$$

(b) Find the percentile of the data value 50?

$$50 \rightsquigarrow \textit{12 th number} \rightsquigarrow \textit{11 numbers below} \rightsquigarrow \frac{11}{26} = 0.423$$

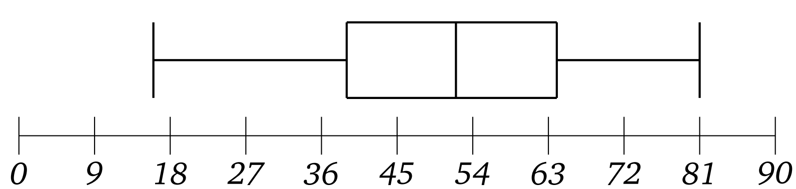
Therefore, 50 is the 42.3 *percentile.*

(c) Find the 5-number summary for this dataset.

$$P_{25} = \frac{25}{100} \cdot 26 = 6.5 \rightsquigarrow 7\textit{th number}$$
$$P_{50} = \frac{50}{100} \cdot 26 = 13 \textit{ (average 13th \& 14th number)}$$
$$P_{75} = \frac{75}{100} \cdot 26 = 19.5 \rightsquigarrow 20\textit{th number}$$

Min	Q_1	*Median*	Q_3	*Max*
16	*39*	*52*	*64*	*81*

(d) Sketch a boxplot for this dataset.



7. (25 points) You ask 5 random Syracuse University students how many times they had high fived Otto the Orange that day. The data is given below.

$$2, \quad 2, \quad 5, \quad 7, \quad 9$$

(a) Find the range and midrange for this dataset.

$$\textit{Range} = \textit{max} - \textit{min} = 9 - 2 = 7$$

$$\textit{Midrange} = \frac{\textit{max} + \textit{min}}{2} = \frac{9+2}{2} = \frac{11}{2} = 5.5$$

(b) Find the mean for the dataset.

$$\overline{x} = \frac{2+2+5+7+9}{5} = \frac{25}{5} = 5$$

(c) Find the variance for this dataset. [You must show your work.]

x_i	$(x_i - \overline{x})^2$
2	*9*
2	*9*
5	*0*
7	*4*
9	*16*
	Sum: 38

Therefore, $s^2 = \frac{1}{n-1}\sum(x-\overline{x})^2 = \frac{38}{4} = 9.5.$

(d) Find the standard deviation for this dataset. [You do not need to show work.]

We know that $s = \sqrt{s^2} = \sqrt{9.5} = 3.08221.$

(e) Construct a 90% confidence interval for the mean number of times a Syracuse University student high fives Otto the Orange per day. [Assume the underlying population is normally distributed.] Be sure to state your conclusions in the context of the problem.

The true population standard deviation is unknown. Therefore, we must use a t-*procedure. This is valid as the underlying population is normally distributed. We have* $n = 5$ *so that we have degrees of freedom* $n - 1 = 4$. *Using a 90% confidence level, we find* $t^* = 2.132$. *Then we have*

$$\begin{array}{ccc} \overline{x} & \pm & t^* \dfrac{s}{\sqrt{n}} \\ 5 & \pm & 2.132 \dfrac{3.08221}{\sqrt{5}} \\ 5 & \pm & 2.94 \end{array}$$

so that we have 90% confidence interval $(2.06, 7.94)$. *Therefore, we are 90% certain that Syracuse University students high five Otto the Orange, on average, between 2.06 and 7.94 times per day.*

(f) Construct a 90% confidence interval for the standard deviation for the number of times a Syracuse University student high fives Otto the Orange per day. [Assume the underlying population is normally distributed.]

We have $s = 3.08221$, $n = 5$, *and degrees of freedom* $n - 1 = 4$. *Using a 90% confidence level, this gives* $\chi^2_R = 9.488$ *and* $\chi^2_L = 0.711$. *Therefore,*

$$\begin{aligned} \frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L} \\ \frac{4 \cdot 3.08221^2}{9.488} < \sigma^2 < \frac{4 \cdot 3.08221^2}{0.711} \\ 4.00507 < \sigma^2 < 53.446 \\ \sqrt{4.00507} < \sigma < \sqrt{53.446} \\ 2.00127 < \sigma < 7.31068 \end{aligned}$$

Therefore, a 90% confidence interval for σ *is* $(2.00, 7.31)$.

8. (20 points) 40% of students at Syracuse University absolutely loved their MAT 121 instructor because they are awesome.

   (a) If 11 students having taken MAT 121 are selected at random, what is the probability that exactly 3 of them loved their instructor?

   $$ {}_{11}C_3(0.40)^3(0.60)^8 = \binom{11}{3}(0.40)^3(0.60)^8 = 0.1774 $$

   (b) If 11 students having taken MAT 121 are selected at random, what is the probability that at most 2 of them loved their instructor?

   $$ 0.0036 + 0.0266 + 0.0887 = 0.1189 $$

   (c) If 11 students having taken MAT 121 are selected at random, what is the probability that at least 3 of them loved their instructor?

   $$ P(\textit{at least 3}) = 1 - P(\textit{at most 2}) = 1 - 0.1189 = 0.8811 $$

   (d) Using the normal approximation, if you surveyed 800 students that took MAT 121, determine the probability that at least 336 of the students loved their instructor.

   We have $800(0.40) = 320 \geq 10$ *and* $800(0.60) = 480 \geq 10$. *Therefore, the normal approximation is appropriate. Using the approximation for proportions, we have* $N(np, \sqrt{np(1-p)}) = N(320, 13.8564)$. *Then we have*

   $$ z_{336} = \frac{336 - 320}{13.8564} = 1.15 \rightsquigarrow 0.8749 $$

   Therefore, the probability is $1 - 0.8749 = 0.1251$.

   (e) Use the continuity correction to improve your answer from (d).

   Using the continuity correction, we find

   $$ z_{335.5} = \frac{335.5 - 320}{13.8564} = 1.12 \rightsquigarrow 0.8686 $$

   Therefore, the probability is $1 - 0.8686 = 0.1314$.

9. (20 points) A not-at-all lonely mathematician counts the number of objects his various cats knock of various surfaces. The counts are given below.

Cat/Surface	Table	Counter	Shelf	Total
Kitty Poppins	3	5	2	10
Jennifurr	4	1	3	8
Brad Kitt	1	2	2	5
Catrick Swayze	3	1	3	7
Fleas Witherspoon	2	3	1	6
Total	13	12	11	36

(a) What is the probability that a cat knocked something off the shelf?

$$\frac{11}{36} = 0.3056$$

(b) What is the probability that Catrick Swayze knocks something off a surface?

$$\frac{7}{36} = 0.1944$$

(c) What is the probability that something is knocked off the counter or that Kitty Poppins knocks something off?

$$\frac{5+1+2+1+3+3+2}{36} = \frac{12+10-5}{36} = \frac{17}{36} = 0.4722$$

(d) Given something was knocked off the table, what is the probability that Fleas Witherspoon did it?

$$\frac{2}{13} = 0.1538$$

10. (10 points) Count the quantities indicated in the following two problems:

(a) How many 'words' can be formed using the letters from the word 'Werbenjagermanjensen'?

$$\frac{20!}{5!4!2!2!2!} = 105,594,705,216,000$$

(b) How many words can be formed from the word 'Felicia' if there must be exactly two letters between the 'F' and the 'c'?

$$\frac{5!}{2!} \cdot 4 \cdot 2! = 480$$

11. (20 points) Count the quantities indicated in the following two problems:

(a) If a club has 16 members, how many ways can they choose a President, Vice-President, and Secretary, assuming no person can hold two or more positions?

$$_{16}P_3 = 16 \cdot 15 \cdot 14 = 3,360$$

(b) How many ways are there of choosing 3 friends to go to a Mumford and Sons concert with if you have 7 friends to choose from?

$$_7C_3 = \binom{7}{3} = 35$$

(c) If you have a room with 5 men and 8 women, how many ways can you select a group consisting of 3 men and 2 women?

$$_5C_3 \cdot {_8C_2} = \binom{5}{3}\binom{8}{2} = 10 \cdot 28 = 280$$

(d) If you have a club with 18 people in it, how many ways can you form *two* committees, one with 5 people and the other with 4 people, each with a designated leader and co-leader? [Assume no one person can be in both committees and no one person can hold both the leader and co-leader positions.]

$$_{18}C_5 \cdot {_5P_2} \cdot {_{13}C_4} \cdot {_4P_2} = 8568 \cdot 20 \cdot 715 \cdot 12 = 1,470,268,800$$

12. (15 points) Out of 50 fanboys surveyed, 37 people answered that liking *Rick and Morty* makes you sophisticated.

   (a) Construct a 95% confidence interval for the proportion of fanboys that think liking *Rick and Morty* makes you sophisticated. Be sure to state your answer in the problem context.

   We have $X = 37$, $n = 50$, *and* $\hat{p} = 37/50 = 0.74$. *Using a 95% confidence interval, we have* $z^* = 1.96$. *Then*

$$\begin{aligned} \hat{p} &\pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ 0.74 &\pm 1.96\sqrt{\frac{0.74(1-0.74)}{50}} \\ 0.74 &\pm 1.96(0.0620) \\ 0.74 &\pm 0.1215 \end{aligned}$$

   This gives 95% confidence interval $(0.6185, 0.8615)$. *Therefore, we are 95% certain that the percentage of* Rick and Morty *fanboys that think that liking* Rick and Morty *makes you sophisticated is between 61.85% and 86.15%.*

   (b) Assuming this sample is a good representation of all fanboys, at least how many fanboys would you have to survey to construct a 95% confidence interval that estimated the proportion of fanboys that think liking *Rick and Morty* makes you sophisticated to within 0.4%?

   We have $z^* = 2.96$ *so that*

$$n \geq \left(\frac{z^*}{m}\right)^2 \hat{p}\,\hat{q} = \left(\frac{1.96}{0.004}\right)^2 0.74(0.26) = 46195.24$$

   so that a sample size of at least $46,196$ *people is required.*

13. (25 points) It was determined that if a woodchuck could chuck wood that the amount of wood it could chuck would be normally distributed with mean 20 ft and standard deviation 3 ft.

(a) Find the probability that a woodchuck could chuck less than 16 ft of wood.

$$z_{16} = \frac{16-20}{3} = -1.33 \rightsquigarrow 0.0918$$

$P(X \leq 16) = 0.0918.$

(b) Find the probability that a woodchuck could chuck more than 25 ft of wood.

$$z_{25} = \frac{25-20}{3} = 1.67 \rightsquigarrow 0.9525$$

$P(X \geq 25) = 1 - 0.9525 = 0.0475$

(c) Find the probability that a woodchuck could chuck between 16 ft and 25 ft of wood.

$$P(16 \leq X \leq 15) = 0.9525 - 0.0918 = 0.8607$$

(d) If 18 woodchucks are chosen at random, what is the probability that the average amount of wood these 18 woodchucks chuck is 22 ft or less.

Because the underlying distribution is normal, the Central Limit Theorem applies, and we can use the sampling distribution. Therefore,

$$z_{22} = \frac{22-20}{3/\sqrt{18}} = \frac{2}{0.707107} = 2.83 \rightsquigarrow 0.9977$$

Therefore, $P(\overline{X} \leq 22) = 0.9977.$

(e) Suppose one did not know that woodchucks could chuck 20 ft of wood on average. How many woodchucks would you have to sample to construct a 90% confidence interval for the average amount of wood they could chuck with an error at most 1 ft.

We know that for a 90% confidence interval, $z^* = 1.645.$ *We have* $m = 1$ *so that*

$$n \geq \left(\frac{z^*\sigma}{m}\right)^2 = \left(\frac{1.645 \cdot 3}{1}\right)^2 = 24.3542$$

so that at least 25 *woodchucks must be used.*