Name: _____ **MAT 121**
Summer 2019 **Ch. 4 Worksheet**

Problem 1: Choose an 'experiment'. For this experiment, give an example of an event and give the sample space.

Problem 2: Suppose you flip a coin twice. What is an example of an event for this situation? What is the sample space? Give some examples of subsets of the sample space.

Problem 3: Suppose you draw a card from a deck of cards. What is an example of an event for this situation? What is the sample space? Give some examples of subsets of the sample space.

Problem 4: What is the Law of Large Numbers. Explain what this means with an example.

Problem 5: 'Translate' these sentences from Mathematics to English:

(a) $P(A)$

(b) $P(\overline{A})$

(c) $P(A \text{ or } B)$

(d) $P(A \cap B)$

Problem 6: Suppose you roll a standard fair die a single time and record the outcome.

(a) What are the possible events?

(b) What is the sample space?

Let A denote the set of even numbers on the die, B denote the set of odd numbers on the die, C denote the collection of the numbers 1 and 2. 'Translate' the following sentences from Mathematics to English or English to Mathematics:

(c) $P(A)$

(d) The probability that the roll was even.

(e) $P(\overline{A})$

(f) The probability that a roll of 1 or 2 was rolled.

(g) The probability that a roll of 3, 4, 5, or 6 was rolled.

Problem 7: Which of the following are probabilities?

$$0 \qquad \frac{5}{7} \qquad -0.10 \qquad 65\% \qquad 50-50 \qquad 125\% \qquad 1.03 \qquad \frac{4}{3} \qquad 1$$

Problem 8: Suppose you are answering a multiple choice question and there are 5 possible answers. What is the probability that you guess correctly? What is the probability of guessing incorrectly?

Problem 9: Suppose you are answering a multiple choice question and there are 5 possible answers. What is the probability that you guess correctly? What is the probability of guessing incorrectly?

Problem 10: Based on a survey by *Cool Mathematicians Today*, 97% of mathematicians believe Mathematics is cool. What is the probability a randomly selected mathematician believes math is 'cool'. What is the probability the mathematician does not believe this?

Problem 11: Suppose you are answering a multiple choice question and there are 8 possible answers, 3 of which are correct. What is the probability that you guess correctly? What is the probability of guessing incorrectly?

Problem 12: Define what it means for events to be disjoint.

Problem 13: True or false: if A is disjoint from B, then B is disjoint from A.

Problem 14: Consider rolling a die. Let A denote the set of even rolls, B denote the set of odd rolls, C denote of rolls larger than 3, and D denote the set of rolls at most 5.

(a) Are A and B disjoint?

(b) Are B and C disjoint?

(c) Are A and C disjoint?

(d) Are C and D disjoint?

Problem 15: Consider flipping a coin 8 times. Let A denote the set of possible outcomes where you obtain at least 1 head, B denote the set of outcomes where you obtain at most 3 tails, C denote the set of possible outcomes where you get at least 7 heads, and D denote the set of outcomes where you obtain at least 3 tails.

(a) Are A and B disjoint?

(b) Are B and C disjoint?

(c) Are A and C disjoint?

(d) Are C and D disjoint?

Problem 16: Define what it means for two events to be independent.

Problem 17: Suppose one is testing patients for a genetic disease. Let A denote the event of testing positive and B the even of testing negative. Are A and B disjoint?

Problem 18: Suppose one stops for coffee on the way to work. Is this event independent of the event "is late for work"?

Problem 19: Suppose one stops for coffee on the way to work. Is this event independent of the event "watched the film *Ex Machina* this week"?

Problem 20: Suppose one is drawing cards from a deck. Let A denote the set of events of drawing a red card, B denote the set of events of drawing a face card, and C denote the set of events of drawing a card with 'value' greater than 8.

(a) Are A and B independent?

(b) Are A and C independent?

(c) Are B and C independent?

Problem 21: What is the notation for conditional probability?

Problem 22: 'Translate' the following sentences from Mathematics to English:

(a) $P(A \mid B)$

(b) $P(B \mid A)$

(c) $P(A \mid B \text{ and } C)$

Problem 23: Consider rolling a die. Let A denote the set of even rolls, B denote the set of odd rolls, C denote of rolls larger than 3, and D denote the set of rolls at most 5. 'Translate' the following from Mathematics to English or English to Mathematics:

(a) $P(A \mid B)$

(b) The probability of rolling a number less than 5 given one rolled an odd.

(c) $P(D \mid C)$

Problem 24: Consider rolling a die. Let A denote the set of even rolls, B denote the set of odd rolls, C denote of rolls larger than 3, and D denote the set of rolls at most 5. Discuss possible independence of these collections of events.

Problem 25: A coin is flipped twice.

(a) Draw an event tree for this experiment.

(b) What is the probability that the first flip is heads?

(c) What is the probability of getting two heads?

(d) What is the probability of getting one head and one tail?

(e) What is the probability of getting at least one head?

Problem 26: A rare genetic disease occurs in only 0.1% of the population but is nearly universally fatal if one has the disease and does not receive treatment. A blood test was developed to screen for the disease. The test is 97% accurate. Suppose a person goes in for testing.

(a) Draw an event tree for this experiment.

(b) What is the probability that the person has the disease?

(c) What is the probability that the person tests negative for the disease?

(d) What is the probability that the person tests positive for the disease given that they have it?

(e) Supposing that the person *does not* have the disease, what is the probability that the test result is positive?

Problem 27: Suppose 95 people are interviewed about where they get their news. Of the people interviewed, 37 get their news from a newspaper, 65 people get their news from television, and 24 people get their news from both.

(a) Draw a complete Venn diagram for this situation.

(b) What is the probability that a person gets their news from the newspaper?

(c) What is the probability that a person gets their news from neither?

(d) What is the probability that a person gets their news from the television, given that they get their news also from the newspaper?

(e) What is the probability that a person who does not watch television gets their news from a newspaper?

(f) If were were not told about the 24 people that receive news from both, could the collection of people that watch the television for their news and the people that get their news from a newspaper been disjoint?

(g) Are the events in (f) independent?

Problem 28: The following table indicates a survey of people, grouped by age, about whether or not they had ever broken a bone.

	0–9	10–19	20–29	30 and over
Broken	7	18	29	30
Never Broken	54	52	45	65

(a) What is the probability that a person has broken a bone?

(b) What is the probability that a person was between 20 and 29 in this study?

(c) Given that a person had broken a bone, what is the probability that they were between 10 and 19?

(d) What is the probability that a person over 30 in this study had broken a bone?

(e) What is the probability that a person chosen at random had broken a bone or was under 10 years old?

Problem 29: Suppose one draws (without replacement in each part) cards from a standard deck of cards.

(a) What is the probability that the card is $\diamondsuit 2$?

(b) What is the probability that one draws a face card?

(c) What is the probability that one draws $\clubsuit 5$ and then $\spadesuit 6$?

(d) Supposing one draws two cards, what is the probability that one draws $\clubsuit 5$ and $\spadesuit 6$?

(e) Supposing one draws a face card, what is the probability that it is the queen of hearts?

(f) I draw a card and tell you that it is red, what is the probability that it is the $\heartsuit 7$?