MAT 121
Summer 2019
Homework 10

"When you look at someone through rose-colored glasses, all the red flags just look like flags."
– Wanda Pierce, BoJack Horseman

Problem 1: According to the Onondaga County Medical Examiner's Office,[1] there were 101 opioid related deaths in the county in 2018. Using the estimate population of Onondaga Country and the estimate number of deaths in the county,[2] we estimate there are 2,913 deaths in Onondaga county.

(a) Using the data above, construct a 95% confidence interval for the proportion of deaths in Onondaga Country caused by opioids. State your conclusions in the context of the problem.

We have $\hat{p} = \dfrac{101}{2913} = 0.0347$ and $z^* = 1.96$. Then

$$
\begin{aligned}
\hat{p} \quad &\pm \quad z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{2913}} \\
0.0347 \quad &\pm \quad 1.96\sqrt{\frac{0.0347(1-0.0347)}{2913}} \\
0.0347 \quad &\pm \quad 0.0066
\end{aligned}
$$

Therefore, we are 95% certain that the true proportion of deaths in Onondaga Country as a result of opioids is between 2.81% and 4.13%.

(b) How many cause of deaths would have to be surveyed to estimate the proportion of deaths in Onondaga Country caused by opioids within 0.1%, assuming you are constructing a 95% confidence interval.

$$
n = \hat{p}(1-\hat{p})\frac{z^{*2}}{m^2} = 0.0347(1-0.0347)\frac{1.96^2}{0.001^2} = 128,677.89
$$

Therefore, at least $128,678$ causes of death need to be surveyed.

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[1]https://healthystories.ongov.net/onondaga-county-opioid-epidemic-data-report/
[2]See http://www.ongov.net/about/ and https://www.kff.org/other/state-indicator/death-rate-per-100000/

Problem 2: You want to estimate parameters (mean and standard deviation) for the distribution of GPAs of Syracuse University students. You survey 12 students and find the following GPAs:

3.54   3.59   3.10   2.89   3.57   2.99   3.48   3.65   3.71   4.00   3.78   3.42

(a) Find the sample mean and sample standard deviation. [You need not show your work.]

We have $\overline{x} = 3.477$ and $s = 0.331$.

(b) Construct a 99% confidence interval for the population mean μ. State your conclusions in the context of the problem.

We have $\overline{x} = 3.477$ and $s = 0.331$. Furthermore, we have degrees of freedom $12 - 1 = 11$ so that $t^* = 3.106$. Then

$$\overline{x} \quad \pm \quad t^* \frac{s}{\sqrt{n}}$$
$$3.477 \quad \pm \quad 3.106 \frac{0.331}{\sqrt{12}}$$
$$3.477 \quad \pm \quad 0.297$$

Therefore, we are 99% certain that the true average GPA of Syracuse University students is between 3.18 and 3.77.

(c) Construct a 99% confidence interval for the population standard deviation σ.

We have $s = 0.331$ and degrees of freedom $12 - 1 = 11$. Therefore, $\chi_L^2 = 2.603$ and $\chi_R^2 = 26.757$. Then

$$\frac{(n-1)\,s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)\,s^2}{\chi_L^2}$$
$$\frac{11(0.331^2)}{26.757} < \sigma^2 < \frac{11(0.331^2)}{2.603}$$
$$0.0450 < \sigma^2 < 0.4630$$
$$0.212 < \sigma < 0.680$$