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MAT 121
Summer 2019
Homework 4

"And that our greatest accomplishments cannot be behind us, because our destiny lies above us."
– Cooper, Interstellar

Problem 1: Explain what is wrong with the following:

(a) $P(A) = 1.1$ *This means the probability of A happening is 110%, which is impossible. Probabilities are always between 0 and 1.*

(b) $P(B) = -0.2$ *This means that the probability of B happening is negative, which is impossible. Probabilities are always between 0 and 1.*

Problem 2: Explain what the Law of Large Numbers says.

The Law of Large Numbers states that the sample probability of an event approaches the actual probability of that event. But the Law of Large Numbers does not say what will happen in any event or how many trials much be performed.

Problem 3: You have flipped a coin 50 times, and it has landed heads up each of these times. This means that the sample probability of heads is 100%. What does the Law of Large Numbers say (or does not say) about there being a tails in the next few flips.

The Law of Large Numbers does not say anything about the next few flips. All it says is that if you continue doing 'enough' flips, the probabilities of heads and tails will eventually stabilize close to their 50-50 chance (though they will continue to vary widely in either direction).

Problem 4: Turn the following mathematical statements into English sentences:

$$A = \text{collection of all blue objects}$$
$$B = \text{collection of all square objects}$$

(a) $\overline{A} =$ *collection of all objects that are not blue.*

(b) $A \cup B =$ *collection of all objects that are blue or square.*

(c) $A \cap B =$ *collection of all objects that are blue and square.*

(d) $P(A) =$ *the probability that an object is blue.*

(e) $P(A \mid B) =$ *the probability that an object is blue given that it is square.*

(f) $P(\overline{A}) =$ *the probability that an object is not blue.*

(g) $P(A \cap \overline{B}) =$ *the probability that an object is blue and not a square.*

Problem 5: Explain the difference between the word 'or' in Mathematics and how it is used in everyday life.

In everyday life, 'or' typically means exclusive or, meaning one or the other but not both. But in Mathematics, 'or' means one or the other or both.

Problem 6: Explain what could be wrong with the following: $P(A) = 0.3$, $P(B) = 0.41$; therefore, $P(A \cup B) = 0.3 + 0.41 = 0.71$.

This assumes that the events A and B are disjoint. If this is the case, then the given probability is correct. Otherwise, this is an overestimate of the probability of A or B.

Problem 7: Explain whether the given events are disjoint or not.

(a) A: winning the lottery; B: the weather being cloudy

These are not disjoint. You can win the lottery on a cloudy day.

(b) A: flying to Vegas; B: driving to Vegas

These events are disjoint. You cannot fly and drive to Vegas at the same time.

(c) A: getting a 'two pair' in cards B: having three red cards in a card game

You can have a 'two pair' and three red cards. The two pair could consist of red cards, you could have one more red card, and all the rest of the cards could be black.

Problem 8: Explain whether the following statement is true or false: $P(A \cap B) = 0$ if A and B are disjoint.

The statement is true. If A and B are disjoint, they have no events in common. Therefore, A and B cannot happen at the same time. But then $P(A \cap B) = 0$.

Problem 9: Explain whether the following statement is true or false: if A and B are independent events, then $P(A \cap B) = 0$.

The statement is false. If $P(A \cap B) = 0$, then A and B are disjoint events. But disjoint events can never be independent.

Problem 10: Do independent events always have to have some event in common? Explain.

Yes. If they did not, then they would be disjoint events. But disjoint events are never independent.

Problem 11: Explain why $0! = 1$ by describing its 'English' interpretation.

$n!$ *is the number of ways of arranging* n *objects in a row. If there are 0 objects, there is one way to do this, i.e. not putting any objects down.*