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MAT 121
Summer 2019
Homework 5

"When I was a boy and I would see scary things in the news, my mother would say to me, 'Look for the helpers. You will always find people who are helping.'"

–Fred Rogers

Problem 1: How many unique ways are there of arranging the letters of the word 'Mississippi'?

Solution. *There are 11 letters with 3 repeats: 's' 4 times, 'i' 4 times, and 'p' twice. So there are*

$$\frac{11!}{4!\,4!\,2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4!\,\cancel{4!}\,2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 34,650$$

Problem 2: A club has 16 members. How many possible outcomes are there if it wishes to elect...

(a) a president, vice president, treasurer, and secretary, assuming no person can hold more than one office.

Order matters and we are choosing 4 positions from 16 members, so this is

$${}_{16}P_4 = 43,680$$

(b) a finance committee with four members.

Order does not matter here and we are choosing 4 positions from 16 members, so this is

$$\binom{16}{4} = 1,820$$

Problem 3: The following table summarizes the numbers of people of various ages and genders who attended a recent movie. If one of these people is chosen at random, what is the probability that the chosen person is either male or in the 0–9 age range?

	0–9	10–19	20–29	30 and up
Female	4	6	24	36
Male	8	12	45	65

Solution. *There are 200 total people, including 130 males and 12 people between the ages of 0 and 9. So we have...*

$$\begin{aligned} P(\textit{male or 0-9 range}) &= P(\textit{male}) + P(\textit{0-9 range}) - P(\textit{male and 9-0 range}) \\ &= \frac{130}{200} + \frac{12}{200} - \frac{8}{200} \\ &= \frac{134}{200} = 0.67 \end{aligned}$$

We could also count the number of males and people between ages 0 and 9: $P(\textit{male or 0-9 range}) = \dfrac{8+12+45+65+4}{200} = \dfrac{134}{200} = 0.67.$

Problem 4: How many ways are there to select a committee of 3 members from among 10 faculty members?

Solution. *Order does not matter. We are selecting 3 people from 10 members, so that we have*

$$_{10}C_3 = 120$$

Problem 5: A survey shows that 60% of university students own laptops. If 8 university students are selected at random, find

(a) the probability that exactly 6 of them own laptops.

The probability that the first 6 have laptops is $(0.6)^6 \cdot (0.4)^2$. *There are a total of* $_8C_6$ *ways of choosing 6 of them to have laptops. Therefore, the probability is...*

$$_8C_6\,(0.6)^6\,(0.4)^2 = 0.20901888$$

(b) the probability that more than 6 of them own laptops.

For more than 6 of them to own laptops, either 7 of them own laptops or 8 of them laptops. Using the logic from (a), the probability that 7 of them own laptops is $_8C_7\,(0.6)^7\,(0.4)^1$ *and the probability that 8 of them own laptops is* $_8C_8\,(0.6)^8\,(0.4)^0$. *Therefore, the probability is*

$$_8C_7\,(0.6)^7\,(0.4)^1 + {}_8C_8\,(0.6)^8\,(0.4)^0 = 0.090 + 0.017 = 0.107$$

Problem 6: A die is rolled 6 times. What is the probability that at least one 5 appeared during these rolls? [Hint: Use compliments.]

Solution. *If* A *is the event that least one 5 appeared during these rolls, then* $\overline{A}$ *is the event that no 5 appeared during these rolls. This means a non-5 must come up on the first roll, and the second roll, and .... But then the probability of this is* $\left(\frac{1}{6}\right)^6$. *Therefore,* $P(\overline{A}) = \left(\frac{1}{6}\right)^6$ *so that we have*

$$P(A) = 1 - P(\overline{A}) = 1 - \left(\frac{1}{6}\right)^6 = 1 - \frac{1}{46656} = \frac{46655}{46656} \approx 0.99997$$

Problem 7: The table below classifies a group of voters according to gender and political affiliation.

	Democrat	Republican	Independent
Male	205	251	33
Female	269	182	57

(a) What is the total number of voters in this group?

$$205 + 269 + 251 + 182 + 33 + 57 = 997$$

(b) If a person is chosen at random from this group of voters, what is the probability of selecting a male or Republican voter.

$$P(\textit{male or Republican}) = \frac{205 + 251 + 33 + 182}{997} = \frac{671}{997} \approx 0.6730$$

(c) What is the probability that a person is a female Democrat?

$$P(\textit{female Democrat}) = \frac{269}{997} = 0.0269$$

(d) What is the probability that a male is independent?

$$P(\textit{Independent} \mid \textit{male}) = \frac{33}{205 + 251 + 33} = \frac{33}{489} \approx 0.0675$$

Problem 8: Suppose you have events A, B, C, where $P(A) = P(B) = P(C) = 0.6$.

(a) Are the events A, B disjoint? Explain.

If A and B were disjoint, then $P(A \cup B) = P(A) + P(B) = 0.6 + 0.6 = 1.2$, which is impossible. Therefore, A and B are not disjoint.

(b) If $P(B \mid A) = 0.9$, are the events A, B independent? Explain.

If A and B were independent, then $P(A \cap B) = P(A)P(B) = 0.6 \cdot 0.6 = 0.36$. But we know $P(A \cap B) = P(A)P(B \mid A) = 0.6 \cdot 0.9 = 0.54 \neq 0.36$. Therefore, A and B cannot be independent.

(c) If events A, C were independent, what is the probability that they occur at the same time?

If A and C are independent, then $P(A \cap C) = P(A)P(C) = 0.6 \cdot 0.6 = 0.36$.

(d) If events A, C were independent, what is the probability that at least one of them occurs?

This is $P(A \cup C)$. But using the previous part, this is $P(A \cup C) = P(A) + P(C) - P(A \cap C) = 0.6 + 0.6 - 0.36 = 0.84$.

Problem 9: At a summer program at a HS, there are 50 students taking part in a STEM headstart program. There are 23 people taking Physics, 31 students taking Chemistry, with 6 students taking both.

(a) Explain how it is possible that there are 23 people taking Physics and 31 people taking Chemistry when there are 50 students in total.

There are some students taking both, i.e. the set of Physics takers and the set of Chemistry takers are not disjoint.

(b) If a student is taken at random, what is the probability that they are taking Physics?

$$P(\textit{Physics}) = \frac{17+6}{50} = \frac{23}{50} = 0.46$$

(c) If a student is taken at random, what is the probability that they are only taking Chemistry?

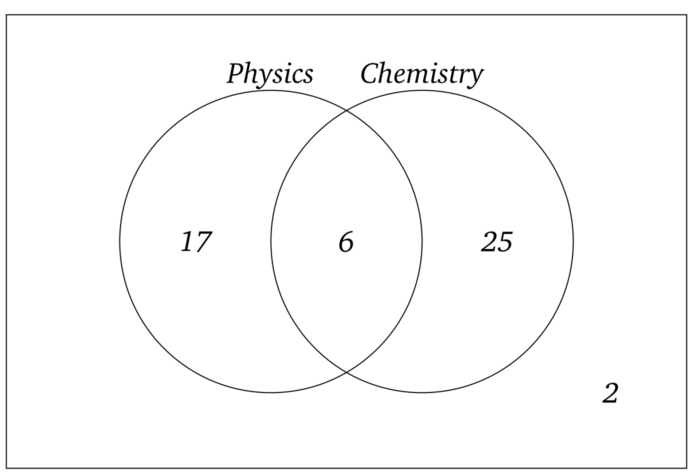
$$P(\textit{Only Chemistry}) = \frac{25}{50} = 0.50$$

(d) If a student is taking Chemistry, what is the probability that they are not taking Physics?

$$P(\textit{Not Physics} \mid \textit{Chemistry}) = \frac{25}{6+25} = \frac{25}{31} \approx 0.806$$

(e) If a student is taken at random, what is the probability that they are taking neither course given that they are taking Physics?

$$P(\textit{Physics} \mid \textit{neither}) = \frac{0}{17+6} = 0$$



Problem 10: A rare genetic disease is discovered and it is known that only 0.01% of the population has the disease. The test for the disease is positive for 99.9% of people that have the disease, while the test is positive for 0.2% of the people who do not have the disease.

(a) What is the probability that any given person tests positive for the disease?

$$0.0000999 + 0.0019998 = 0.0020997$$

(b) What is the probability that a person tests positive and has the disease?

$$0.0000999$$

(c) What is the probability that a person who has the disease tests positive?

$$0.999$$

(d) What is the probability that a person who tests positive actually has the disease?

$$\frac{0.0000999}{0.0000999 + 0.0019998} = \frac{0.0000999}{0.0020997} = 0.0475782$$