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MAT 121
Summer 2019
Homework 6

"Live by this credo: have a little laugh at life and look around you for happiness instead of sadness. Laughter has always brought me out of unhappy situations."
–Red Skelton

Problem 1: Let A and B denote events in a sample space with $P(A) = 0.3$, $P(B) = 0.8$, and $P(A \cap B) = 0.4$.

(a) Are the events A and B disjoint?

No. For one, $P(A \cap B) = 0.4 \neq 0$*. Moreover, if they were disjoint, then* $P(A \text{ or } B) = P(A) + P(B) = 0.3 + 0.8 = 1.1$*, which is impossible.*

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.8 - 0.4 = 0.7$

(c) $P(\overline{B}) = 1 - P(B) = 1 - 0.8 = 0.2$

(d) Are A and B independent? Explain.

If A *and* B *were independent, then* $P(A \cap B) = P(A)P(B)$*. But* $P(A \cap B) = 0.4$ *and* $P(A)P(B) = 0.3 \cdot 0.8 = 0.24$*. Therefore,* A *and* B *cannot be independent.*

(e) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.4}{0.8} = 0.50$

Problem 2: Given two independent events A, B with $P(A) = 0.7$ and $P(B) = 0.4$, find. . .

(a) $P(A \text{ and } B) = P(A)P(B) = 0.7 \cdot 0.4 = 0.28$

(b) $P(B \mid A) = P(B) = 0.4$

(c) $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.4 - 0.28 = 0.82$

Problem 3: You are performing a study to examine the relationship between sports viewership and (STEM) employment. Based on the following sample, answer the following questions:

	Watch Sports	Does Not Watch Sports
Unemployed	34	44
STEM Job	15	51
Non-STEM Job	31	43

(a) How many total people were involved in the study?

$$34 + 44 + 15 + 51 + 31 + 43 = 218$$

(b) How many people did not watch sports or had a STEM job?

$$44 + 51 + 43 + 15 = 153$$

(c) What is the probability that person with a STEM job watches sports?

$$\frac{15}{218} = 0.069$$

(d) What is the probability that a person watches sports or has a non-STEM job?

$$\frac{34 + 15 + 31 + 43}{218} = \frac{123}{218} = 0.564$$

(e) What is the probability that a person has a STEM job?

$$\frac{15 + 51}{218} = \frac{66}{218} = 0.303$$

(f) What is the probability that a person with a STEM job watches sports?

$$\frac{15}{15 + 51} = \frac{15}{66} = 0.227$$

Problem 4: If the odds against something, let's say an event A, are $27\colon 1$, what is $P(A)$? What is $P(\overline{A})$?

If the odds are $27\colon 1$*, then every 28 times, you expect one 'success'. Therefore,* $P(A) = 1/28$*. This means that* $P(\overline{A}) = 1 - P(A) = 27/28$*.*

Problem 5: If you roll two die, what is the probability that the sum of the numbers is 6? What is the probability that the sum is 2? What about the sum being 1?

There are 36 possibilities for the numbers on the dice. Then we only need count the ones that sum to the given number. This gives

$$P(6) = \frac{5}{36} = 0.139$$
$$P(2) = \frac{1}{36} = 0.028$$
$$P(1) = \frac{0}{36} = 0$$

Problem 6: How many 6 digit passwords can be made from the digits 0–9? How many that start with a 4? How many that are even?

There are $10^6 = 1,000,000$ *(10 choices for each of the 6 digits) total passwords. There are* $1 \cdot 10^5 = 100,000$ *(1 choice for the first spot, and 10 for each subsequent spot) total passwords beginning with a 4. There are* $10^5 \cdot 5 = 500,000$ *(10 choices each for the first 5 spots, but only 5 (0,2,4,6,7) for the last spot.*

Problem 7: A bag contains four coins—three real and one trick coin. The real coins are all ordinary coins with a head and a tail. The trick coin has two heads. Suppose a person is blindfolded, reaches into the bag, and flips the coin. They then remove the blindfold and records what side of the coin faces up.

(a) What is the probability that a heads is observed?

$$P(heads) = \frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8} = 0.625$$

(b) What is the probability that a tails is observed?

$$P(tails) = 1 - P(heads) = 1 - \frac{5}{8} = \frac{3}{8} = 0.375$$

(c) Supposing they grabbed the trick coin, what is the probability that a heads was observed?

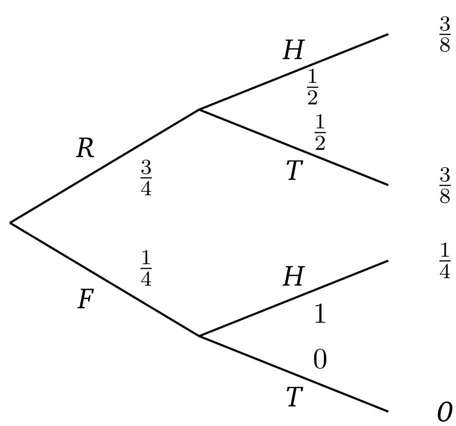
$$P(Heads \mid Fake) = 1$$

(d) Supposing they grabbed one of the real coins, what is the probability that a tail was observed?

$$P(Tails \mid Real) = \frac{1}{2}$$

(e) Given that the coin was heads up, what is the probability that the fake coin was selected?

$$P(Fake \mid Heads) = \frac{1/4}{5/8} = \frac{2}{5} = 0.40$$



Problem 8: Ads suggest that 64% of homes for sale in a certain area have garages, 21% have swimming pools, and 4% of homes have just a swimming pool.

(a) Find the probability that a home has a swimming pool or garage.

$$P(\textit{Pool or Garage}) = 47\% + 17\% + 4\% = 68\% = 0.68$$

(b) Find the the probability that a home has neither a swimming pool nor a garage.

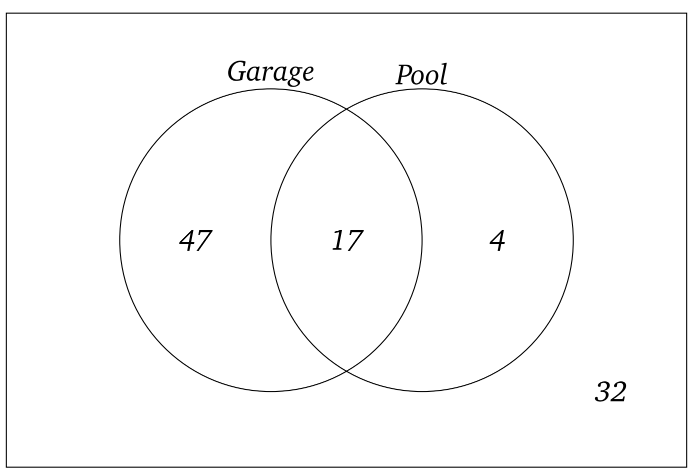
$$P(\textit{Neither}) = 32\% = 0.32$$

(c) Find the probability that a home only has a garage.

$$P(\textit{Garage}) = 47\% = 0.47$$

(d) Find the probability that a home has a swimming pool if it has a garage.

$$P(\textit{Pool} \mid \textit{Garage}) = \frac{17\%}{47\% + 17\%} = \frac{17\%}{64\%} = 0.2656$$



Problem 9: You have 5 friends that you want to take to an SU basketball game. But they do not all get along so you should keep the groups small. You get a discount if you buy 4 tickets, one for you and three for your friends. How many times can you go to the games always bringing 3 friends and never having the same 3 friends there with you? If you go that many times, how many times will one of your friends not go to the game with you and your other friends?

The order in which they go does not matter and because they do not go more than once, there is no repetition. All that matters is the collection of 3 friends is not the same. Therefore, the total number of times is ${}_7C_3 = 10$. *Now you always go. Choose a friend's perspective. They are always going with you. But they are also going with two other friends. If they group is to be the same, these other two people they are going with is the same. But these groups can never repeat. Therefore, the number of times they go is the number of ways of choosing these other 2 friends from the (remaining) 4 friends, i.e.* ${}_4C_2 = 6$. *They either go or not. Therefore, they do not go* $10 - 6 = 4$ *times.*

Problem 10: The letters of the word 'braids' are rearranged to create new 'words'. Creating all the possible words and putting them in ordinary alphabetic order, what number in the list is the word 'baidrs'? [Hint: Count them alphabetically. How many words begin with 'a'? How many with begin with 'b'? How many begin with 'ba'? So on. . .]

There are $6! = 720$ *total 'words'. The first words must begin 'a' but then the rest can be anything. There are* $1 \cdot 5! = 120$ *such words. Alphabetically, the next words begin with 'b', same as our word. So the first letter requirements have been met. All the next words also have the second letter 'a', so the second letter is also satisfied. But all the words after this have third letter 'd', not 'i', so they come first. There are* $1 \cdot 1 \cdot 1 \cdot 3! = 6$ *such words. But then the next work must begin with 'baidrs', which is our word. Therefore, it must occur after the* $120 + 6 = 126$*th word, i.e. word 127.*

Problem 11: How many ways can the letters of the word 'permutations' be arranged so that there 4 letters between the 'p' and the 's'? [Hint: Fix 'p' and 's'. How many arrangements 'around' these letters? Take note of the double 't'. How many placements for 'p' and 's'?]

Imagine the letter spaces as blanks. No matter where the 'p' and 's' are placed, there are 10 other letters to place—each giving a different 'word.' So for each possible placement of the 'p' and 's', there are $\frac{10!}{2!} = 1,814,400$ *total words. [Note, it would be* $10!$ *but the 't's are repeated.] If there is to be a gap of 4 between the 'p' and 's', working left to right, we see there are only 7 ways of doing this. But for each such placement, either 'p' or 's' will come first, i.e. there are two choices. Therefore, the total number of ways of doing this are* $1814400 \cdot 7 \cdot 2 = 25,401,600$. *Fun fact, if you could write one of these combinations down each second and wrote without stopping, it would take 294 days to write them all.*

Problem 12: How many ways can the letters of the word 'baclava' be rearranged so that the letters 'a', 'l', and 'a' all appear together in some order? [Hint: Treat the letters needing to appear together as one giant letter, being careful that there is more than one way to 'block' them together.]

The word 'baclava' has 7 letters. If the letters 'a', 'l', and 'a' must appear together, then fixing them in some spot, there are only 4 spots remaining with 4 distinct letters. There are $4! = 24$ *ways of arranging these other letters, each giving a distinct 'word'. Now because 'a', 'l', and 'a' must appear in a row, working from left to right, there are only 5 possible placements for these three letters. Once placed (as one giant block), you may rearrange the letters 'a', 'l', and 'a' in any order. The number of distinct orderings of these (each giving a unique 'word') is* $\frac{3!}{2!} = 3$. *[Notice we have to consider the repeated 'a'.] This gives* $24 \cdot 5 \cdot 3 = 360$ *total possible words.*

A standard deck of playing cards has 52 cards, consisting of 4 suits of cards: hearts, clubs, diamonds, and spades, each suit containing 13 cards. The suits hearts and diamonds are colored red while the suits clubs and spades are colored black. The cards types are 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace—a total of 13 types. A typical game of poker is a card game where the player is dealt 5 cards. For this game, a full house is a hand which contains 3 of one type of cards, i.e. 2's, Jacks, etc., and 2 cards of another type. For example, a full house could consist of 2 of hears, 2 of clubs, 2 of diamonds, and an ace of hearts with an ace of diamonds. A three of a kind is where one holds 3 cards of one type in their hand.

Problem 13: What is the probability of a full house?

A hand consists of 5 cards from a deck of 52 cards. Order does not matter. Therefore, there are ${}_{52}C_5 = 2,598,960$ *total possible hands. We need to choose a card for the three of a kind. There are 13 choices, i.e.* ${}_{13}C_1$. *Once the choice is made, say 3's. There are four 3's to choose from. This can be done in* ${}_4C_3 = 4$ *ways. Then we must select the number of ways of getting a pair. But we can no longer get a pair from the three of a kind type, because there is only one card of that type left. Then we have 12 choices, i.e.* ${}_{12}C_1$. *From that type, there are 4 cards, of which we must choose 2. There are* ${}_4C_2 = 6$ *ways of doing this. Therefore, the probability is*

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}} = \frac{13 \cdot 4 \cdot 12 \cdot 6}{2598960} = \frac{3744}{2598960} = 0.00144058 = 0.144058\%$$