Name: $\qquad$
MAT 121
Summer 2019
Homework 7
"We often refuse to accept an idea merely because the tone of voice in which it has been expressed is unsympathetic to us." -Friedrich Nietzsche

Problem 1: A fair game consists of spinning a wheel with the numbers 1 through 6 on it, though not all the numbers are equally likely. If the spinner lands on an even number, you must pay $\$ 2$. If the number lands on a 3 or 5 , you win or lose nothing, but if the spinner lands on a 1 , you win $\$ 5$.

| Amount | $-\$ 2$ | $\$ 0$ | $\$ 5$ |
| :---: | :---: | :---: | :---: |
| Probability | 0.50 | 0.35 | 0.15 |

(a) Fill in the probability for winning $\$ 5$ in the table above to make the table a probability distribution.
(b) What is the average amount a player expects to win per game in the long run?

$$
\mu=\sum x P(x)=-2(0.50)+0(0.35)+5(0.15)=-\$ 0.25
$$

(c) Based on the information from (b), should one play this game? Explain.

No. The expected payout is negative. So no matter the starting capital, in the long run, you will eventually lose all your money.
(d) Compute the standard deviation, $\sigma$, for the probability distribution. Show your work.

| $x$ | $x-\mu$ | $(x-\mu)^{2}$ | $(x-\mu)^{2} P(x)$ |
| ---: | ---: | ---: | ---: |
| -2 | -1.75 | 3.0625 | 1.53125 |
| 0 | 0.25 | 0.0625 | 0.021875 |
| 5 | 5.25 | 27.5625 | 4.134375 |
|  |  |  | Total: 5.69 |

So we have $\sigma^{2}=5.69$. Therefore, the standard deviation is $\sigma=\sqrt{ } 5.69=2.39$.

Problem 2: An economic wellness survey is being conducted on a city to determine its funding needs. The analysis team determines that for this city, incomes are approximately normally distributed with mean $\mu=40.1$ and standard deviation $\sigma=15.7$, measured in tens of thousands of dollars.
(a) What percent of people make less than $\$ 50,000$ a year?

$$
z_{50}=\frac{50-40.1}{15.7}=\frac{9.9}{15.7}=0.63 \rightsquigarrow 0.7357=73.57 \%
$$

(b) What percent of people make more than $\$ 50,000$ a year?
$P($ more than 50,000$)=1-P($ at most 50,000$)=1-0.7357=0.2643=26.43 \%$
(c) What percent of people make more than $\$ 80,000$ a year?

$$
z_{80}=\frac{80-40.1}{15.7}=\frac{39.9}{15.7}=2.54 \rightsquigarrow 0.9945
$$

Therefore, $P($ greater than 80,000$)=1-P($ at most 80,000$)=1-0.9945=0.0055=0.55 \%$.
(d) What percent of people make between $\$ 50,000$ and $\$ 80,000$ a year?
$P($ between 50,000 and 80,000$)=P($ at most 80,000$)-P($ at most 50,000$)=0.9945-0.7357=0.2588$

