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MAT 121
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Homework 9

"Success is 1% inspiration, 98% perspiration, and 2% attention to detail."
–Phil Dunphy, Modern Family

Problem 1: Dyslexia is the most common type of learning disorder and is characterized by reading difficulties. It is estimated that between 5–15% of the US population suffers from some form of dyslexia. Suppose that approximately 11% of the school aged children in a certain state have dyslexia. A school board wants to estimate the number of dyslexic students they will have in their schools to estimate the amount of money they need to allot to academic assistance resources. The board knows that their school district has 8,435 students.

(a) What is the probability that more than 850 students will suffer from some form of dyslexia?

We have $np = 8435(0.11) = 927.85 \geq 10$ *and* $n(1-p) = 8435(0.89) = 7507.15 \geq 10$. *Therefore, we can use the normal approximation,* $N(np, \sqrt{np(1-p)}) = N(927.85, 28.7365)$. *Therefore,*

$$z_{850} = \frac{850 - 927.85}{28.7365} = \frac{-77.85}{28.7365} = -2.71 \rightsquigarrow 0.0034$$

Therefore, the probability is $1 - 0.0034 = 0.9966$.

(b) What is the probability that less than 950 students will suffer from some form of dyslexia?

We have $np = 8435(0.11) = 927.85 \geq 10$ *and* $n(1-p) = 8435(0.89) = 7507.15 \geq 10$. *Therefore, we can use the normal approximation,* $N(np, \sqrt{np(1-p)}) = N(927.85, 28.7365)$. *Therefore,*

$$z_{950} = \frac{950 - 927.85}{28.7365} = \frac{22.15}{28.7365} = 0.77 \rightsquigarrow 0.7794$$

Therefore, the probability is 0.7794.

(c) What is the probability that between 850 and 950 students will suffer from some form of dyslexia?

Using (a) and (b), we have probability $0.9966 - 0.0034 = 0.9932$.

(d) Use the continuity correction to improve your answer from (a).

$$z_{849.5} = \frac{849.5 - 927.85}{28.7365} = \frac{-78.35}{28.7365} = -2.73 \rightsquigarrow 0.0032$$

Therefore, the probability is $1 - 0.0032 = 0.9968$.

(e) What is the probability that more than 12% of the students will suffer from some form of dyslexia?

We have $np = 8435(0.11) = 927.85 \geq 10$ *and* $n(1-p) = 8435(0.89) = 7507.15 \geq 10$. *Therefore, we can use the normal approximation,* $N(p, \sqrt{\frac{p(1-p)}{n}}) = N(0.11, 0.0034)$. *Then*

$$z_{0.13} = \frac{0.13 - 0.12}{0.0034} = \frac{0.01}{0.0034} = 2.94 \rightsquigarrow 0.9984$$

Therefore, the probability is $1 - 0.9984 = 0.0016$.

Problem 2: Stardoes is a coffee company that sells millions of cups of coffee a day. They are designed a new multiuse measuring cup that can be used to make a large variety of their drinks. The cup needs to have a mark exactly 12.7 cm from the base of the cup to indicate the fill line for a large cup of coffee. They do not want the line lower (to avoid more lawsuits involving underfilled drinks) or higher to avoid 'giving away' coffee. Using a sample of 57 cups, they find an average marked height of 12.9 cm. The machines are known to mark within an accuracy of $\sigma = 0.08$ cm.

(a) Construct a 91% confidence interval for the average marked line height. State your conclusions in the context of the problem.

We have $57 \geq 30$ *so that the Central Limit Theorem applies. We have* $\overline{x} = 12.9$ *cm and* $\sigma = 0.08$ *cm. We have* $z^* = 1.695$*. Then*

$$\begin{aligned} \overline{x} &\pm z^* \frac{\sigma}{\sqrt{n}} \\ 12.9 &\pm 1.695 \frac{0.08}{\sqrt{57}} \\ 12.9 &\pm 0.0180 \end{aligned}$$

Therefore, we are 91% certain that the true mean marked height is between 12.882 *cm and* 12.918 *cm.*

(b) How many cups must be sampled to be sure that the average marked line height is approximated within 0.01 cm if the confidence level is 95%.

$$\begin{aligned} n &\geq \left(\frac{z^* \sigma}{m} \right)^2 \\ n &\geq \left(\frac{1.96 \cdot 0.08}{0.01} \right)^2 \\ n &\geq (15.68)^2 \\ n &\geq 245.862 \end{aligned}$$

Therefore, at least 246 cups must be surveyed.