Applied Problems:
Arclength & Curvature

"DNA is like a computer program but far, far more advanced than any software ever created."

-Bill Gates, The Road Ahead

DNA

Deoxyribonucleic acid (DNA) is a molecule consisting of two polynucleotide chains that coil around each other, forming a double helix. This helix contains the genetic information necessary for most forms of life (along with Ribonucleic acid, or RNA). The strands consist of simpler monomeric units called nucleotides—each composed of four nitrogen containing nucleobases: cytosine, guanine, adenine, and thymine. The helix structure allows DNA to split and be rematched with new half-helix strains of DNA to form another new full chain. This allows cells to multiply while still containing the same essential DNA. The replication of DNA in animals is so accurate that, on average, there is only one mistake for every 10 billion nucleotides replicated.

To give an idea of the scale of this accuracy, if you were to write every word in every major world language (over 80 languages) down 1,000 times, to achieve the same accuracy, you could misspell at most 2 of those words. The study of DNA is essential for DNA testing, medical treatments, Bioinformatics, DNA editing, etc. The radius of a human DNA helix is approximately 10 angstroms, i.e. 1 nanometer (10^{-9} meters)—though some measure DNA to be slightly larger. To complete one complete turn in the helix, DNA rises about 34 angstroms. Each strand of DNA contains about 2.9×10^8 turns in a single helix.



Problem:

- (a) Find a parametrization for a single helix within a DNA molecule.
- (b) Use your answer from (a) to find the length of a strand of DNA.
- (c) If you could pull this DNA strand taunt, how long would it be? Give a representation of scale that an 'average person' could understand. [There are over 2.76 quadrillion strands of DNA in a human.]
- (d) Compute the curvature of a molecule of DNA.
- (e) Reconcile (c) and (d).