Name: *Caleb McWhorter— Solutions*

MAT 397— Fall 2020
Applied Problems:
Chain Rule

"Childhood obesity issue is critically important to me because it's critically important to the health and success of our kids, and of this nation, ultimately."

–Michelle Obama

Body Mass Index (BMI)

The chain rule is particularly useful when there may not be exact equations describing the variables for a function. Starting with an 'arbitrarily' defined function, one can still compute the rate of change of the function with respect to the partials of its component functions. These individual rates of change can then be empirically measured and applied to find the rate of change of the original function, even when the rate of the change of the components is not explicitly known. We will look at an example of this.

Obesity is a common but serious disease. Obesity related conditions: heart disease, stroke, type 2 diabetes, and cancer, are the leading cause of death in the United States. Moreover, they are the leading causes of preventable, premature death. Obesity related care is not only expensive and draining for the patient, but cost the United Stated over one hundred billion dollars a year.

Health professionals measure the health of a persons body mass using a weighted function comparing their weight against their height. The most common weighted measurement is body mass index (BMI). BMI gives a more meaningful assessment of weight because a person that weights 190 lbs and is 5'0 is likely not as healthy as someone who is 190 lbs and is 6'1". The BMI of an individual depends on their weight (in pounds), w, and height h (in inches), and is given by

$$\text{BMI} = \frac{703w}{h^2}$$

[Note, there is no 703 if w and h are measured in kg and m, respectively.] The following gives the distribution of weight class by a persons BMI. Suppose you are a pediatrician. One of your patients

BMI	**Weight Class**
0.0 – 15.9	Severely Underweight
16.0 – 18.5	Underweight
18.5 – 24.9	Normal Weight
25.0 – 29.9	Overweight
30.0 – 39.9	Obese
40.0+	Morbidly Obese

was brought in by their parents the previous year for a regular checkup. During last years checkup, the child was 11 years old and was 84.2 lb and 56.5 in. You now measure the child to be 108 lb and 58.1 in

Problem:

(a) What was the child's BMI last year?

(b) What is the child's BMI this year?

(c) Using the chain rule, find the rate of change of BMI in terms of h, w, dw/dt, and dh/dt.

(d) Using (c) and the given information about the child, should you be concerned about the child's BMI? Explain.

Solution.

(a)

$$\text{BMI} = \frac{703(84.2)}{56.5^2} = 18.5$$

Note this places the child in the normal weight category (but barely).

(b)

$$\text{BMI} = \frac{703(108)}{58.1^2} = 22.5$$

Note this places the child in the normal weight category.

(c)

$$\begin{aligned}\frac{d}{dt}(\text{BMI}) &= \frac{\partial(\text{BMI})}{\partial w}\frac{dw}{dt} + \frac{\partial(\text{BMI})}{\partial h}\frac{dh}{dt} \\ &= \frac{703}{h^2}\frac{dw}{dt} - \frac{1406w}{h^3}\frac{dh}{dt}\end{aligned}$$

(d) We have $w = 108$, $h = 58.1$, $\frac{dw}{dt} \approx \frac{108 \text{ lb} - 84.2 \text{ lb}}{1 \text{ year}} = 23.8$ lb/yr, and $\frac{dh}{dt} \approx \frac{58.1 \text{ in} - 56.5 \text{ in}}{1 \text{ year}} = 1.6$ in/yr. So

$$\frac{d(\text{BMI})}{dt} \approx \frac{703}{58.1^2}(23.8) - \frac{1406(108)}{58.1^3}(1.6) = 4.96 - 1.24 = 3.72 \text{ BMI/yr}$$

While the child is in the normal weight category currently, at this rate of change, their BMI next year will be 26.22, which would place them in the overweight category. The child, perhaps, should be monitored an encouraged to continue good diet and exercise over the next year.