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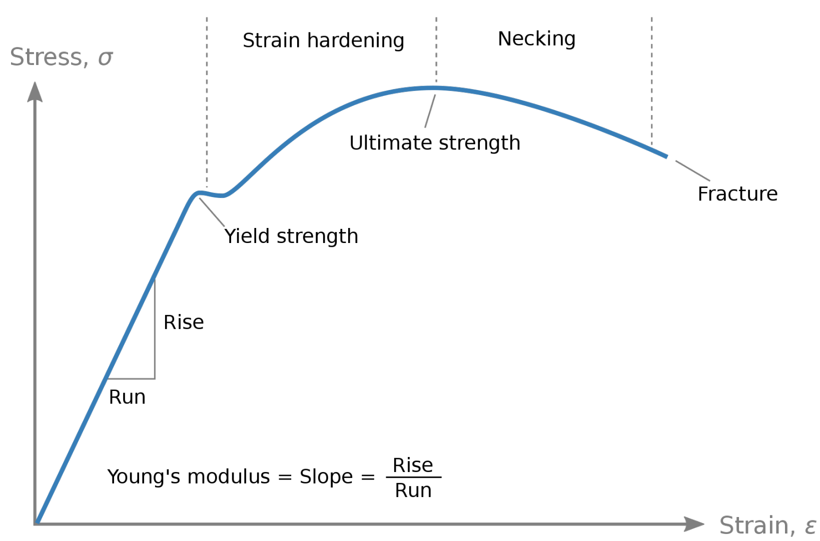
MAT 397— Fall 2020
Applied Problems: Vectors

"Failure is an inevitability. Every scientist was told, "No," over and over. The ones we remember, the ones who changed our lives: the Curies, the Salks, the Barnards, they're the ones who wouldn't take "No" for an answer. Failure is inevitable, unavoidable. But failure should never get the last word. You have to hold on to what you want. You have to take "No" for an answer, and take what's coming to you. Never give in. Never give up. Stand up. Stand up and take it."
–Meredith Grey, Grey's Anatomy

Tension & Catastrophic Failure

Vectors are the natural language with which to describe forces. They allow computation of net force vectors from objects being acted on from the sides, top and bottom, or any other angle with ease. Vectors also allow simple computation of strain forces. Classic examples are tension problems. Tension is a 'maintaining' force. For instance when you tow an object with a car, the force the attaching lead 'feels' to keep the load in tow is a tension force. Tension forces are also present in a rope during a tug of war, cables holding up objects, bent surfaces supporting weight, etc.

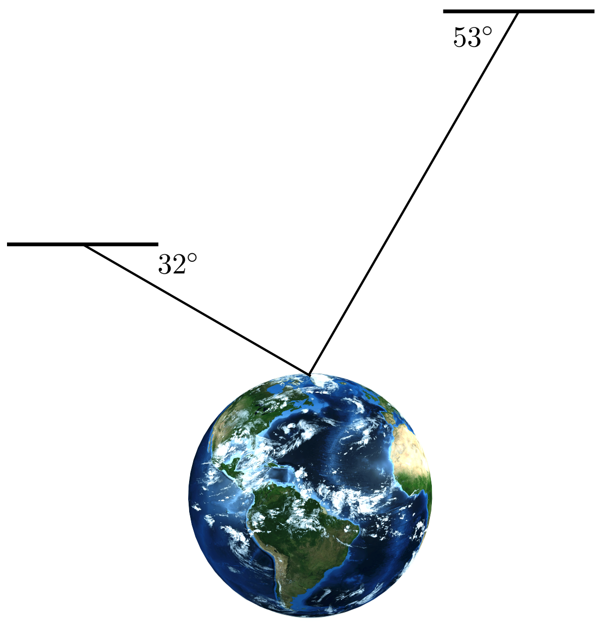
These tension forces are a natural starting point to analyze failure in materials. As materials experience greater and greater force (for example in the form of tension) they will begin to yield to the force by bending, twisting, stretching, etc. These failures are measured by the materials elasticity and deformation. Generally, these are characterized using a few common numbers: proportionality limit, yield strength, and yield point.



The proportionality limit is the point up to which stress is proportional to strain (according to Hooke's Law) so that the stress-strain plot is a straight line with gradient the elastic modulus of the material. The yield strength (or elastic limit because up to this point the stress-strain is elastic) is the point at which a specific amount of plastic deformation (permanent deformation) occurs. This

Plot from https://en.wikipedia.org/wiki/Stress%E2%80%93strain_curve amount can be taken to be any percentage of the unstressed length but is commonly taken to be 0.1%–0.2% of the unstressed length of the material. Finally, the yield point or ultimate strength of the material is the point where the material cannot withstand further increase in applied stress and catastrophic failure and fracture is imminent.

Suppose you are helping in the design and construction of a Natural History museum. The museum wants to suspend a large globe from two viewing platforms in the main hall. This way the visitors will be able to view the globe from below and above. The museum's main hall has already been constructed, and the globe will be constructed by a separate contractor. Given the desired globe location and the current platform placement, the globe will be hung by two wire ropes making angles of $32°$ and $53°$ with their ceiling anchors, respectively. [See the diagram below.]



Problem:

(a) Let the weight of the globe be W. Find the tension *and* tension vectors for each wire in terms of W. [Recall that weight is the mass times the acceleration due to gravity, i.e. $W = mg$.]

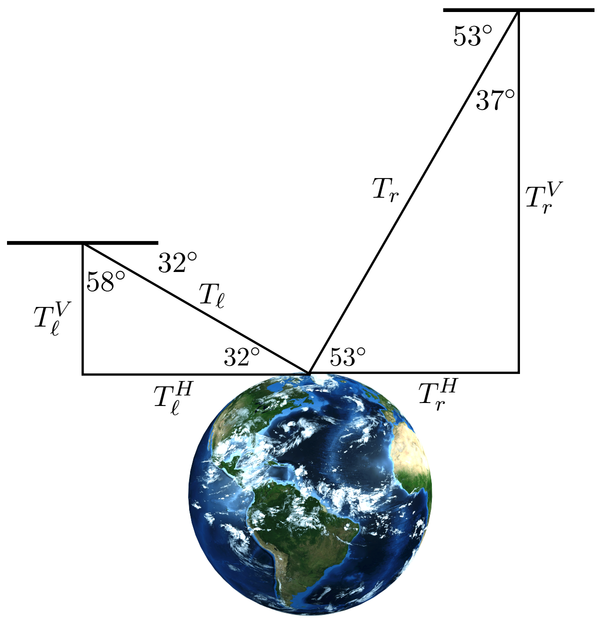
(b) So long as nothing is in contact with the wire ropes and everything is static, tension in the wire ropes will be constant. The weakest point, in principle, is the thinnest slice of the wire rope. Assume the wire rope for both sides has been ordered and is 1/2 in (12.7 mm) Bright wire, uncoated, fiber core (FC) wire rope, improved plow steel (IPS). According to Engineering Toolbox,[1] the ultimate strength is 85,600 psi (pounds-force per square inch), or equivalently 590.2 MPa (megapascals, note 1 MPa$=10^6$ Pa and 1 Pa$=1$ N/1 m^2). It also gives a safe load as 17,120 psi, or equivalently 118 MPa.

What will be your recommended weight to the contracting company for the globe? What is the minimum 'guaranteed unsafe' weight for the globe?

(c) What are some other factors that you could affect all these measurements and recommendations?

[1] https://www.engineeringtoolbox.com/wire-rope-strength-d_1518.html

(a) Let T_ℓ be the tension vector on the left and T_r be the tension vector on the right. Letting V denote vertical components and H represent horizontal components, setting the origin where the wires are anchored to the globe, we have the following diagram.



The only force in the vertical direction is due to gravity pulling on the globe. Because the globe is not moving, the upwards force of the cables must match the downwards force of gravity. Using this fact and trigonometry, we have

$$\begin{aligned} W &= T_\ell^V + T_r^V \\ &= |T_\ell| \sin 32^\circ + |T_r| \sin 53^\circ \end{aligned}$$

There are no external forces in the horizontal direction, and the amount of force on the globe from the left and right wires must be the same because the globe is not moving. This gives

$$\begin{aligned} T_\ell^H &= T_r^H \\ |T_\ell| \cos 32^\circ &= |T_r| \cos 53^\circ \\ |T_\ell| \cos 32^\circ - |T_r| \cos 53^\circ &= 0 \end{aligned}$$

Putting all this information together, we have

$$\begin{aligned} W &= |T_\ell| \sin 32^\circ + |T_r| \sin 53^\circ \\ 0 &= |T_\ell| \cos 32^\circ - |T_r| \cos 53^\circ \end{aligned}$$

Solving for $|T_\ell|$ and $|T_r|$, we find that

$$\begin{aligned} |T_\ell| &= \frac{W \cos 53^\circ}{\sin 32^\circ \cos 53^\circ + \sin 53^\circ \cos 32^\circ} \approx 0.604114\, W \\ |T_r| &= \frac{W \cos 32^\circ}{\sin 32^\circ \cos 53^\circ + \sin 53^\circ \cos 32^\circ} \approx 0.851287\, W \end{aligned}$$

To find T_ℓ and T_r as vectors, observe that

$$T_\ell = -T_\ell^H\, \mathbf{i} + T_\ell^V\, \mathbf{j} = -|T_\ell| \cos 32^\circ\, \mathbf{i} + |T_\ell| \sin 32^\circ\, \mathbf{j} = \langle -0.512318W, 0.320132W \rangle = W \langle -0.512318, 0.320132 \rangle$$

$$T_r = T_r^H\, \mathbf{i} + T_r^V\, \mathbf{j} = |T_r| \cos 53^\circ\, \mathbf{i} + |T_r| \sin 53^\circ\, \mathbf{j} = \langle 0.512317W, 0.679868W \rangle = W \langle 0.512317, 0.679868 \rangle$$

(b) The wire will have area cross section $A = \pi r^2 = \pi(0.25 \text{ in})^2 = 0.19635 \text{ in}^2$ or 0.000126677 m^2. Then the pressure in the wire ropes at each thinnest slice will be

$$\text{Pressure}_\ell = \frac{0.604114W \text{ lb}}{0.19635 \ \text{in}^2} = 3.07672W \text{ psi}$$
$$\text{Pressure}_r = \frac{0.851287W \text{ lb}}{0.19635 \ \text{in}^2} = 4.33556W \text{ psi}$$

Or equivalently,

$$\text{Pressure}_\ell = \frac{0.604114gW \text{ N}}{0.000126677 \ \text{m}^2} \cdot \frac{1}{10^6} = 0.0467355W \text{ MPa}$$
$$\text{Pressure}_r = \frac{0.851287gW \text{ N}}{0.000126677 \ \text{m}^2} \cdot \frac{1}{10^6} = 0.0658574W \text{ MPa}$$

where $g \approx 9.8 \text{ m/s}^2$. Setting these to 85,600 psi or 590.2 MPa, respectively, we can find the maximum weight each cable can support. This gives 27821.8 lb for the left and 19743.7 lb for the right. Equivalently, this gives 12628.5 kg for the left and 8961.79 kg for the right. [These are the same up to issues of rounding.]

Therefore, the most the globe can weight is 19,743.7 lb. Of course, this is a maximum weight. We can redo the computations above for the recommended safe load for the cables. We then find a recommended safe load of 5564.37 lb for the left and 3948.74 lb for the right, or equivalently, 2524.85 kg for the left and 1791.75 kg for the right. Therefore, the recommended safe weight for the globe is 3948.74 lb.

(c) Some factors which could affect these computations and their validity are: whether the cables are anchored at the same point on the globe, whether any other forces will act on the globe, e.g. wind, whether the platforms can support the weight, whether the fact that we have not considered the cable to have weight will play a large factor, whether there are other construction considerations, e.g. earthquake or other safety construction guidelines, whether the globe will be able to rotate (and hence twist the cables), etc.