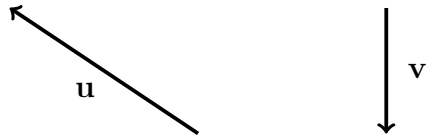
MAT 397: Exam 1 **Name:** _____
Fall – 2020
09/21/2020
80 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	14	
2	16	
3	16	
4	16	
5	16	
6	16	
7	16	
Total:	110	

1. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors given below.



As accurately as possible, sketch the following vectors:

(a) (3 points) $-\frac{1}{2}\mathbf{v}$

(b) (3 points) $\mathbf{u} + \mathbf{v}$

(c) (4 points) $\mathbf{v} - \mathbf{u}$

(d) (4 points) $\mathbf{proj}_{\mathbf{v}}\,\mathbf{u}$

2. Suppose you are given the following:

$$\begin{aligned} \mathbf{u} &= \langle 2, -1, 3 \rangle & \|\mathbf{v}\| &= 3\sqrt{5} \\ \mathbf{v} &= \langle 4, -2, 5 \rangle & \mathbf{u} \cdot \mathbf{v} &= 25 \\ \mathbf{w} &= \langle 1, -1, 1 \rangle & \mathbf{u} \times \mathbf{w} &= \langle 2, 1, -1 \rangle \end{aligned}$$

(a) (3 points) Find $\|\mathbf{u}\|$.

(b) (3 points) Is $\mathbf{u}$ parallel to $\mathbf{v}$? Justify your answer completely.

(c) (5 points) Is $\mathbf{u}$ perpendicular to $\mathbf{w}$? Justify your answer.

(d) (5 points) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

3. Suppose you are given the following:

$$\begin{aligned} \mathbf{u} &= \langle 2, -1, 3 \rangle & \|\mathbf{v}\| &= 3\sqrt{5} \\ \mathbf{v} &= \langle 4, -2, 5 \rangle & \mathbf{u} \cdot \mathbf{v} &= 25 \\ \mathbf{w} &= \langle 1, -1, 1 \rangle & \mathbf{u} \times \mathbf{w} &= \langle 2, 1, -1 \rangle \end{aligned}$$

(a) (4 points) Without explicitly computing the angle between $\mathbf{u}$ and $\mathbf{v}$, determine whether the angle between $\mathbf{u}$ and $\mathbf{v}$ is acute, right, or obtuse. Justify your answer.

(b) (4 points) Are the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ coplanar? Justify your answer.

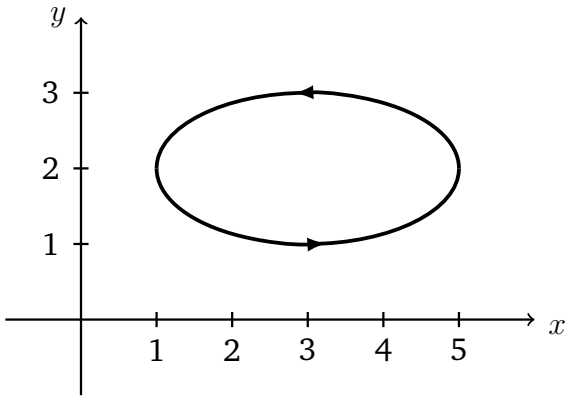
(c) (4 points) Compute the projection $\mathbf{proj}_{\mathbf{v}}\, \mathbf{u}$.

(d) (4 points) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{w}$.

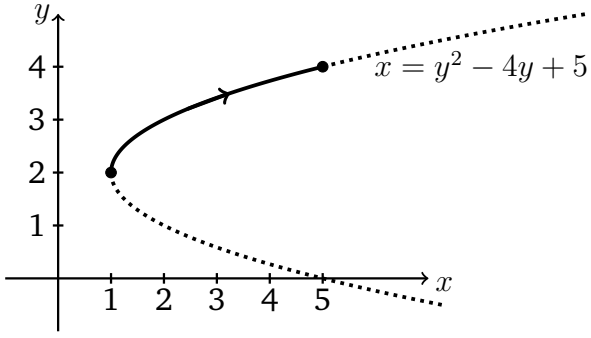
4. Parametrize the following curves:

   (a) (6 points) the directed line segment from $(-2, 3, 0)$ to $(1, -1, 2)$.

   (b) (5 points) the ellipse shown below.



   (c) (5 points) the directed portion of the curve between the two given points shown below.



5. Complete the following parts:

   (a) (8 points) If $\mathbf{r}'(t) = 3t^2\,\mathbf{i} - e^{1-t}\,\mathbf{k}$ and $\mathbf{r}(1) = 2\,\mathbf{j} + \mathbf{k}$, find $\mathbf{r}(t)$.

   (b) (8 points) If $\mathbf{x}(t) = \langle 1, \sqrt{t}, \ln t \rangle$, find an integral which computes the length of the curve between $\mathbf{x}(1)$ and $\mathbf{x}(8)$. *Do not evaluate the integral.*

6. Complete the following parts:

   (a) (8 points) Evaluate the following limit:

$$\lim_{(x,y)\to(-2,1)} \frac{x^2 + 3xy + 2y^2}{x^2 - 4y^2}$$

   (b) (8 points) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^5 + 4y^5}$$

7. Complete the following parts:

   (a) (8 points) Find the equation of the line perpendicular to the plane $x + 2y - z = 6$ at the point where the plane intersects the y-axis.

   (b) (8 points) Find the equation of the plane containing the following two lines: $\ell_1(t) = (5t + 4, t - 1, 5t + 2)$ and $\ell_2(t) = (3 - 5t, 2 - t, 1 - 5t)$.