MAT 397: Exam 2 **Name:** ____________________

Fall – 2020

10/19/2020

150 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 12 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total:	200	

1. Consider the surface given by $y^2 + xy + 2 = x^3 - y^4 + z^5$.

   (a) (10 points) Find the equation of the tangent plane to this surface at the point $(1, 0, 1)$.

   (b) (10 points) Find the parametric equations for the normal line to the surface at the point $(1, 0, 1)$.

2. Let $f(x, y, z) = x \cos y + 3z$.

   (a) (4 points) Find $f(2, 0, 1)$.

   (b) (8 points) Find the total differential of $f(x, y, z)$.

   (c) (8 points) Use parts (a) and (b) to approximate $f(2.1, -0.1, 0.8)$.

3. Define the following functions:

$$\begin{aligned} f(x,y) &= xy + y^3 \\ x(t) &= \sqrt[3]{t^2} \\ y(s,t) &= e^{t/s} \end{aligned}$$

Use the multivariable chain rule to find the following in terms of x, y, s, t:

(a) (10 points) $\dfrac{\partial f}{\partial s}$

(b) (10 points) $\dfrac{\partial f}{\partial t}$

4. Let $f(x, y) = xe^{x-2y}$ and $\mathbf{u} = 2\,\mathbf{i} + \mathbf{j}$.

   (a) (8 points) Find $D_{\mathbf{u}}f(2, 1)$.

   (b) (3 points) What is the direction of the maximum rate of increase for $f(x, y)$ at $(2, 1)$? What is the rate of change in that direction?

   (c) (3 points) What is the direction of the maximum rate of decrease for $f(x, y)$ at $(2, 1)$? What is the rate of change in that direction?

(d) (3 points) Estimate the change in $f(2,1)$ if you travel a distance of 0.5 in the direction of $\mathbf{u}$.

(e) (3 points) At the point $(2,1)$, give a direction in which $f(x,y)$ does not change, i.e. a direction for which $D_{\mathbf{v}}f(2,1) = 0$.

5. (20 points) Show that the only critical values for $f(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$ are $(0, 0, 0)$ and $(2, 0, 0)$. Classify these two critical values.

6. (20 points) Use the method of Lagrange Multipliers to find the points on the ellipse $3x^2 - 4xy + 3y^2 = 50$ nearest and farthest from the origin. *[Hint: Work instead with the square of the distance. You should have four points in total.]*

7. (20 points) Integrate the following:

$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx$$

8. (20 points) Write down an integral in cylindrical coordinates to compute the volume of the region inside the sphere centered at the origin with radius $\sqrt{2}$ and above the plane $z = 1$. *You do not need to evaluate the integral.*

9. (20 points) Consider the following integral:

$$\iiint_R (x+z)\, dV$$

where R is the region in the first octant bounded above by the sphere centered at the origin with radius 3 and below by the cone $z^2 = x^2 + y^2$. Set-up this integral in spherical coordinates. *You do not need to evaluate the integral.*

10. Consider an object whose shape is given by the region in Quadrants I & II bounded by the curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$, and whose mass density at a point (x, y) is given by $D(x, y) = \dfrac{1}{x^2 + y^2}$. Let $(\overline{x}, \overline{y})$ denote the center of mass.

    (a) (4 points) Explain why $\overline{x} = 0$.

    (b) (8 points) Write down an integral in polar coordinates that gives the total mass of the object. *You do not need to evaluate the integral.*

    (c) (8 points) Write down an integral in polar coordinates that gives $\overline{y}$. *You do not need to evaluate the integral.*