**Bootcamp: Prepare Yourself for Calculus III**

This packet is to help you refresh on previously learned material that will be essential in Calculus III. In addition, it will serve as a resource for you throughout the semester to help ensure you have a solid foundation on which to build your Calculus III knowledge. Please keep in mind that this packet is designed to help you review and refresh on many previously learned topics, but there may be other topics not included in this packet that you also will be expected to know. **However, if you ever find yourself struggling, please seek help.** There are a plethora of resources available to help you be successful. These resources include:

1. The Calculus Help Center (Carnegie 102): http://math.syr.edu/undergraduate/math-help.html
2. The Center for Learning & Student Success (Lower Level of Bird Library): http://class.syr.edu/
3. Your T.A.
4. Your instructor

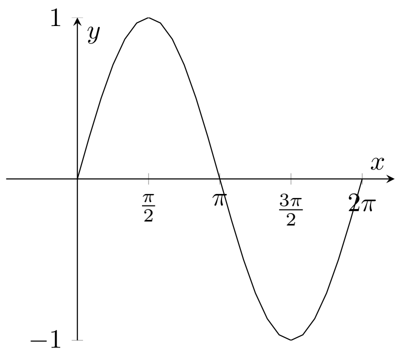
# Contents

# 1 Common Graphs

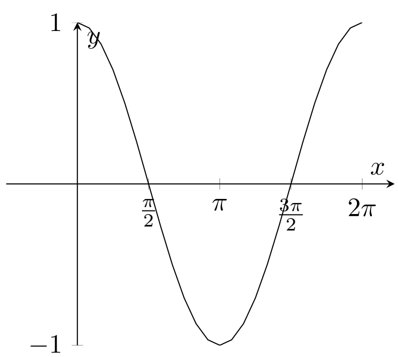
This section details a list of common graphs that you should know. Please review and famliarize yourself with them as you may need to recall them at various times throughout the semester as well as subsequent Calculus courses.

## 1.1 Trigonometric Graphs

1. $y = \sin x$
   a. This is a continuous, periodic function.
   b. Its domain is $(-\infty, \infty)$.
   c. This is an odd function, i.e. $\sin(-x) = -\sin x$




2. $y = \cos x$
   a. This is a continuous, periodic function.
   b. Its domain is $(-\infty, \infty)$.
   c. This is an even function, i.e. $\cos(-x) = \cos x$



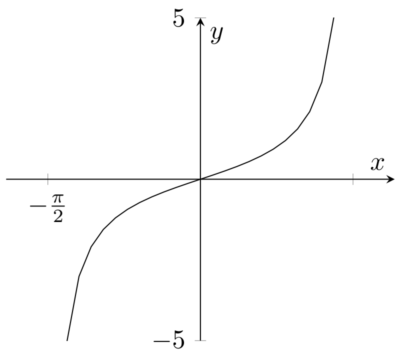
3. $y = \tan x$

a. This is a periodic function.

b. Its domain is all real numbers except $x = \dfrac{\pi}{2} + n\pi$, where $n$ is an integer.

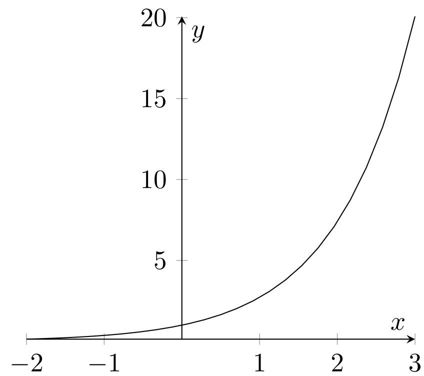
c. It is continuous on its domain.

d. It has vertical asymptotes where $\cos x = 0$ i.e. at $x = \dfrac{\pi}{2} + n\pi$, where $n$ is an integer.

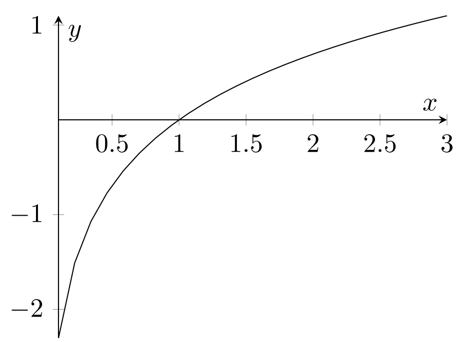


## 1.2 $y = e^x$ and $y = \ln x$

1. $y = e^x$
   a. This is a continuous function.
   b. Its domain is $(-\infty, \infty)$.
   c. It is a one-to-one function.



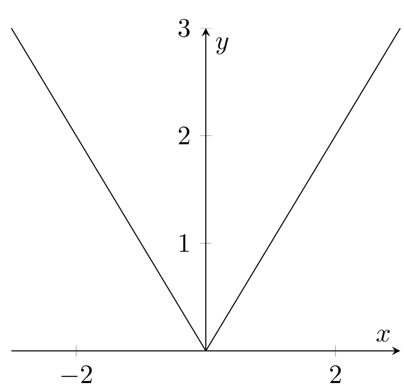

2. $y = \ln x$
   a. This is a continuous function.
   b. Its domain is $(0, \infty)$.
   c. It is a one-to-one function.




## 1.3 The Absolute Value Function

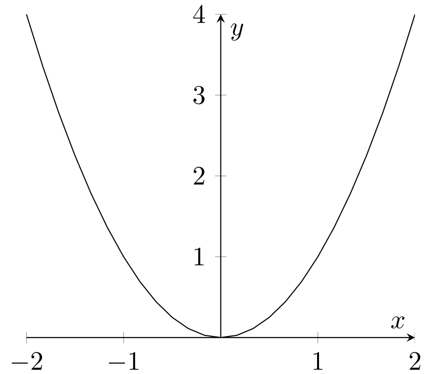
1. $y =| x$.
   a. This is a piece-wise defined function. $y =| x = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

   b.This is a continuous function.
   c. Its domain is $(-\infty, \infty)$.

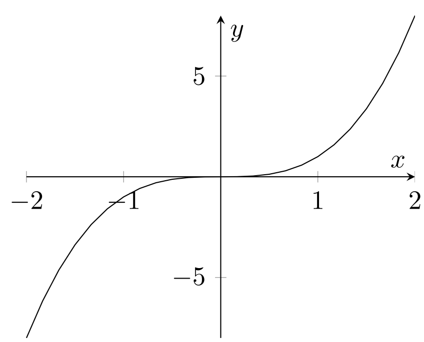


## 1.4 Algebraic Functions

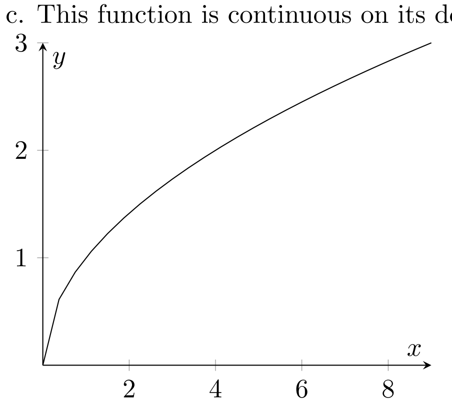
1. $y = x^2$
   a. This is a continuous function.
   b. Its domain is $(-\infty, \infty)$.
   c. This is an even function.




2. $y = x^3$
   a. This is a continuous function.
   b. Its domain is $(-\infty, \infty)$.
   c. This is an odd function.

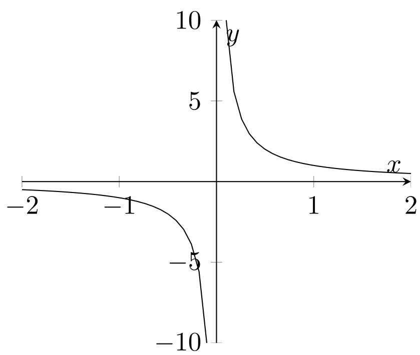



3. $y = \sqrt{x}$
   a. Its domain is $[0, \infty)$.
   c. This function is continuous on its domain.



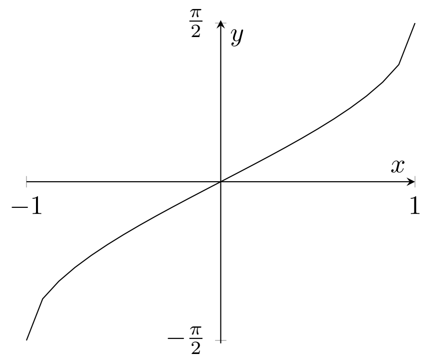
4. $y = \dfrac{1}{x}$

a. Its domain is $(-\infty, 0) \cup (0, \infty)$.
b. This function is continuous on its domain.

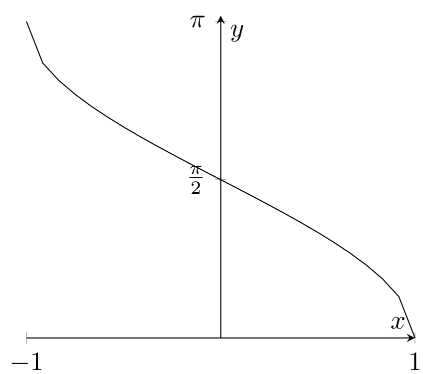


## 1.5 Inverse Trig Functions

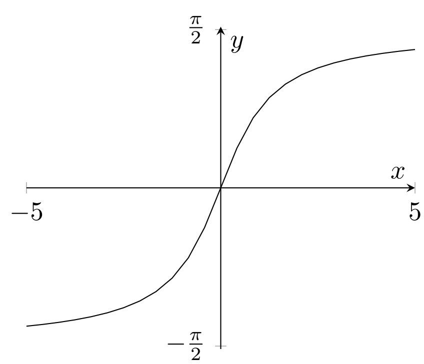
1. $y = \sin^{-1} x = \arcsin x$
   a. Its domain is $[-1, 1]$.
   b. Its range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.




2. $y = \cos^{-1} x = \arccos x$
   a. Its domain is $[-1, 1]$.
   b. Its range is $\left[0, \pi\right]$.




3. $y = \tan^{-1} x = \arctan x$
   a. Its domain is $(-\infty, \infty)$.
   b. Its range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



# 2 Trigonometric Identities

This section details commonly used trigonometric identities.

## 2.1 Reciprocal Identities

1. $\csc x = \dfrac{1}{\sin x}$
2. $\sec x = \dfrac{1}{\cos x}$
3. $\cot x = \dfrac{1}{\tan x}$

## 2.2 Quotient Identities

1. $\tan x = \dfrac{\sin x}{\cos x}$
2. $\cot x = \dfrac{\cos x}{\sin x}$

## 2.3 Pythagorean Identities

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $1 + \cot^2 x = \csc^2 x$

## 2.4 Double Angle Identities

1. $\sin(2x) = 2\sin x \cos x$
2. $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

## 2.5 Half-Angle Identities

1. $\sin^2 x = \dfrac{1}{2}(1 - \cos(2x))$
2. $\cos^2 x = \dfrac{1}{2}(1 + \cos(2x))$

# 3 Lines

This section reviews the equations of lines and properties of lines.

## 3.1 Forms of Equations of Lines

1. General Form: $Ax + By = C$ for constants, $A, B,$ and $C$.

2. Point-Slope Form: $y - y_1 = m(x - x_1)$, where $m$ is the slope of the line and $(x_1, y_1)$ is a point on the line.

3. Slope-Intercept Form: $y = mx + b$, where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept.

4. Vertical Lines: $x = a$.

5. Horizontal Lines: $y = b$.

## 3.2 Facts about Lines

1. The domain of linear functions is $(-\infty, \infty)$.
2. A vertical line is not a function.
3. Parallel lines have the same slope.
4. Perpendicular lines have negative reciprocal slopes.
5. A horizontal line is perpendicular to a vertical line.

# 4 Complex Fractions

## 4.1 Simplifying Complex Fractions

Method #1:

1. Create a single fraction in the numerator and a single fraction in the denominator.
2. Apply the division rule for fractions, i.e. multiply the numerator by the reciprocal of the denominator.
3. Simplify.

Method #2:

1. Find the Least Common Denominator (LCD) of all denominators in the complex fraction.
2. Multiply the numerator by the LCD and multiply the denominator by the LCD.
3. Simplify.

Examples

1. Simplify the complex fraction using the above described method #1. $\dfrac{4+\frac{1}{x}}{3+\frac{2}{x^2}}$.

Solution: $\dfrac{4+\frac{1}{x}}{3+\frac{2}{x^2}} = \dfrac{\frac{4x}{x}+\frac{1}{x}}{\frac{3x^2}{x^2}+\frac{2}{x^2}} = \dfrac{\frac{4x+1}{x}}{\frac{3x^2+2}{x^2}} = \dfrac{4x+1}{\cancel{x}} \cdot \dfrac{x^{\cancel{2}}}{3x^2+2} = \dfrac{x(4x+1)}{3x^2+2} = \dfrac{4x^2+x}{3x^2+2}$

2. Simplify the complex fraction using the above described method #2. $\dfrac{4+\frac{1}{x}}{3+\frac{2}{x^2}}$.

Solution: $\dfrac{4+\frac{1}{x}}{3+\frac{2}{x^2}} = \left(\dfrac{4+\frac{1}{x}}{3+\frac{2}{x^2}}\right) \cdot \left(\dfrac{x^2}{x^2}\right) = \dfrac{4x^2+\frac{x^{\cancel{2}}}{\cancel{x}}}{3x^2+\frac{2\cancel{x^2}}{\cancel{x^2}}} = \dfrac{4x^2+x}{3x^2+2}$

## 4.2 Exercises

1. $\dfrac{3-\frac{1}{2x}}{5-\frac{4}{x^3}}$

2. $\dfrac{\frac{3}{x+1}-\frac{2}{x-1}}{\frac{5}{x-1}-\frac{3}{x+1}}$

# 5 Completing the Square

The goal of completing the square is to rewrite a quadratic function of the form $y = ax^2 + bx + c$ into the form $y = a(x-h)^2 + k$.

## 5.1 Method

Method: How to Complete the Square:

1. Put parentheses around the "$x$" terms, i.e. $y = \left(ax^2 + bx\right) + c$.

2. Factor $a$ out of the parentheses, i.e. $y = a\left(x^2 + \frac{b}{a}x\right) + c$.

3. Compute $\frac{1}{2}\left(\frac{b}{a}\right)$ and then square your result, i.e. $\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2$.

4. Add your result from step #3 inside the parentheses and subtract $a$ times this result outside to keep your equation balanced! That is, $y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$.

5. Write the result from Step #4 in the form $y = a(x-h)^2 + k$.

Examples: Complete the square to rewrite the function in the form $y = a(x-h)^2 + k$.

1. $y = x^2 - 4x + 1$

   Solution:

   $y = \left(x^2 - 4x\right) + 1$; now for step #2 $a = 1$ so move to step #3. $\left(\frac{-4}{2}\right)^2 = 4$

   $y = \left(x^2 - 4x + 4\right) + 1 - 4 = (x-2)^2 - 3$, therefore $\boxed{y = (x-2)^2 - 3}$

2. $y = 5x^2 - 4x - 2$

   Solution:

   $y = 5x^2 - 4x - 2 = \left(5x^2 - 4x\right) - 2 = 5\left(x^2 - \frac{4}{5}x\right) - 2$; so for step #3: $\left(\frac{1}{2}\left[\frac{-4}{5}\right]\right)^2 = \left(\frac{-2}{5}\right)^2 = \frac{4}{25}$

   $y = 5\left(x^2 - \frac{4}{5}x + \frac{4}{25}\right) - 2 - 5\left(\frac{4}{25}\right) = 5\left(x - \frac{2}{5}\right)^2 - 2 - \frac{4}{5} = 5\left(x - \frac{2}{5}\right)^2 - \frac{14}{5}$

   therefore, $\boxed{y = 5\left(x - \frac{2}{5}\right)^2 - \frac{14}{5}}$

## 5.2 Exercises

Complete the square to rewrite the function in the form $y = a(x-h)^2 + k$.

1. $y = x^2 + 6x + 11$
2. $y = -x^2 + 5x - 32$
3. $y = 2x^2 + 8x + 23$
4. $y = -3x^2 + 6x - 14$
5. $y = 2x^2 + x - 7$

# 6 Logarithms

## 6.1 Properties of Logarithms

Definition: A function $f$ is called a one-to-one function if it never takes on the same value twice; that is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

The Horizontal Line Test: A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Definition: Let $f$ be a one-to-one function with domain $A$ and range $B$. Then its inverse function, denoted $f^{-1}(x)$, has domain $B$ and range $A$ and is defined by $f^{-1}(y) = x \iff f(x) = y$ for any $y$ in $B$.

Since $y = a^x$ is a one-to-one function, it has an inverse which is called a logarithmic function with base $a$. $\log_a x = y \iff a^y = x$

Notes:
1. The most convenient base for exponential functions in calculus is $e$, the inverse of this is the natural logarithm, i.e. the logarithm with base $e$. The natural log is denoted by $\ln x$.

2. When no subscript is written it is understood the base is 10. This is called the common logarithm.

3. Change of Base Formula: For any positive number $a(a \neq 1)$, we have $\log_a x = \dfrac{\ln x}{\ln a}$

Examples: Evaluate.

1. $\log_2 16$

   Solution:
   $\log_2 16 = y \Rightarrow 2^y = 16 \because 16 = 2^4$ so $y = 4 \therefore \boxed{\log_2 16 = 4}$

2. $\log_4 \dfrac{1}{64}$

   Solution:
   $\log_4 \dfrac{1}{64} = y \Rightarrow 4^y = \dfrac{1}{64} = \dfrac{1}{4^3} = 4^{-3}$ so $y = -3 \therefore \boxed{\log_4 \dfrac{1}{64} = -3}$

Inverse Properties of Exponential and Logarithmic Functions:

1. $\log_a (a^x) = x$ for every $x \in \mathbb{R}$.

2. $a^{\log_a x} = x$ for every $x > 0$.

Examples: Evaluate.

1. $\log_2 2^{5x+9}$

   Solution:
   Using Inverse Property #1, $\log_2 2^{5x+9} = \boxed{5x + 9}$

2. $\log 10^{x^2+3x+1}$

   Solution:
   Using Inverse Property #1, $\log 10^{x^2+3x+1} = \boxed{x^2 + 3x + 1}$

Properties of the Logarithms: If $x$ and $y$ are positive numbers and $r$ is any real number,

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r\log_a x$

Examples:
#1-2 Use the properties of logarithms to expand the quantity.

1. $\log_3(x^5\sqrt[3]{9x+8})$
   Solution:

   $\log_3(x^5\sqrt[3]{9x+8}) = \log_3 x^5 + \log_3\sqrt[3]{(9x+8)}$, using property # 1

   $\log_3 x^5 + \log_3\sqrt[3]{(9x+8)} = 5\log_3 x + \frac{1}{3}\log_3(9x+8)$; using property #3

   $\therefore \boxed{\log_3(x^5\sqrt[3]{9x+8}) = 5\log_3 x + \frac{1}{3}\log_3(9x+8)}$

2. $\ln\left(\frac{x(2x^2+5)^3}{\sqrt{5x^4+7}}\right)$
   Solution:

   $\ln\left(\frac{x(2x^2+5)^3}{\sqrt{5x^4+7}}\right) = \ln x + \ln(2x^2+5)^3 - \ln\sqrt{5x^4+7}$; using properties #1 & #2

   $\ln x + \ln(2x^2+5)^3 - \ln\sqrt{5x^4+7} = \ln x + 3\ln(2x^2+5) - \frac{1}{2}\ln(5x^4+7)$; using property #3

   $\therefore \boxed{\ln\left(\frac{x(2x^2+5)^3}{\sqrt{5x^4+7}}\right) = \ln x + 3\ln(2x^2+5) - \frac{1}{2}\ln(5x^4+7)}$

#3-4 Use the properties of logarithms to rewrite each expression as the sum, difference, and/or multiple of logarithms.

3. $2\ln(x^2+6) + 3\ln(5x+4) - \frac{2}{3}\ln(x-7)$
   Solution:

   $2\ln(x^2+6) + 3\ln(5x+4) - \frac{2}{3}\ln(x-7) = \ln(x^2+6)^2 + \ln(5x+4)^3 - \ln(x-7)^{2/3}$; using property #3

   $\ln(x^2+6)^2 + \ln(5x+4)^3 - \ln(x-7)^{2/3} = \ln\left((x^2+6)^2(5x+4)^3\right) - \ln(x-7)^{2/3}$; using property #1

   $\ln\left((x^2+6)^2(5x+4)^3\right) - \ln(x-7)^{2/3} = \ln\left(\frac{(x^2+6)^2(5x+4)^3}{(x-7)^{2/3}}\right)$; using property #2

   $\therefore \boxed{2\ln(x^2+6) + 3\ln(5x+4) - \frac{2}{3}\ln(x-7) = \ln\left(\frac{(x^2+6)^2(5x+4)^3}{(x-7)^{2/3}}\right)}$

4. $3\left(2\log_2(9x+1)+\log_2(x+3)\right)-\frac{1}{2}\log_2(x^2+1)$

Solution:

$$3\left(2\log_2(9x+1)+\log_2(x+3)\right)-\frac{1}{2}\log_2(x^2+1)=6\log_2(9x+1)+3\log_2(x+3)-\frac{1}{2}\log_2(x^2+1)$$

$$6\log_2(9x+1)+3\log_2(x+3)-\frac{1}{2}\log_2(x^2+1)=\log_2(9x+1)^6+\log_2(x+3)^3-\log_2(x^2+1)^{1/2}; \text{ by property \#3}$$

$$\log_2(9x+1)^6+\log_2(x+3)^3-\log_2(x^2+1)^{1/2}=\log_2\left((9x+1)^6(x+3)^3\right)-\log_2(x^2+1)^{1/2}; \text{ by property \#1}$$

$$\log_2\left((9x+1)^6(x+3)^3\right)-\log_2(x^2+1)^{1/2}=\log_2\left(\frac{(9x+1)^6(x+3)^3}{(x^2+1)^{1/2}}\right); \text{ by property \#2}$$

$$\therefore \boxed{3\left(2\log_2(9x+1)+\log_2(x+3)\right)-\frac{1}{2}\log_2(x^2+1)=\log_2\left(\frac{(9x+1)^6(x+3)^3}{(x^2+1)^{1/2}}\right)}$$

## 6.2 Exercises

#1-4 Evaluate.

1. $\ln 1$

2. $\ln e$

3. $\log 100$

4. $\log_2\left(\frac{1}{4}\right)$

#5-8. Simplify each expression.

5. $\ln e^6$

6. $\ln e^{10x^3+7}$

7. $\log 10^{3x}$

8. $\log_3 3^{\sqrt{x+1}}$

#9-12 Use the properties of logarithms to rewrite each expression as the sum, difference, and/or multiple of logarithms.

9. $\log\left(x^3\sqrt[4]{5x^2+9}\right)$

10. $\log_5\left(\frac{(x^2+1)^6(1-x^3)^7}{(x^2+x+1)^9}\right)$

11. $\ln\left(\frac{\sqrt{x^4+9}}{(x^3+7x)^5\sqrt[3]{2x+1}}\right)$

12. $\ln\sqrt[5]{\left(\frac{(5x+9)^4(x-8)^3}{(7x+11)^2}\right)}$

#13-16 Use the properties of logarithms to rewrite the expression as a single logarithm.

13. $4\ln(x+8)+2\ln(x^5+9)-\frac{1}{2}\ln(3x-7)$

14. $2\Big[\ln(2x+11)+\ln(x^7+1)\Big]-5\ln(x-9)$

15. $\frac{1}{3}\log_2(5-x)-4\log_2(5x+8)-3\log_2(x^3+2)$

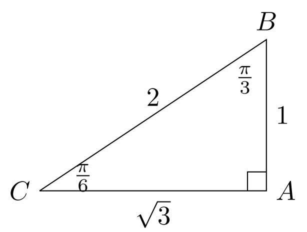
16. $\frac{2}{3}\log(9x+1)-5\Big[\log(x+2)+\log(x+3)\Big]$

# 7 Evaluating Trigonometric Functions

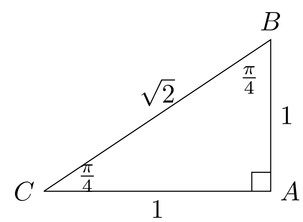
## 7.1 Reference Angles

Definition: A reference angle is the acute angle formed by the terminal side of the angle $t$ and the horizontal axis.

Recall: The 30-60-90 Right Triangle




Recall: The 45-45-90 Right Triangle




How to Determine a Reference Angle: To determine the reference angle of:
1. an angle in Quadrant II it would be $\pi - t$.
2. an angle in Quadrant III it would be $t - \pi$.
3. an angle in Quadrant IV it would be $2\pi - t$.
Note: If your angle is less than 0 or greater than $2\pi$ add or subtract $2\pi$ as many times as you need in order to determine an equivalent, coterminal angle that is between 0 and $2\pi$.

Examples: Find the reference angle of:

1. $\frac{5\pi}{3}$

   Solution:
   $\frac{5\pi}{3}$ is in Quadrant IV. Therefore, the reference angle is $2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3} = \boxed{\frac{\pi}{3}}$

2. $\frac{17\pi}{6}$

   Solution:
   Notice, $\frac{17\pi}{6} > 2\pi$. Therefore, first determine a coterminal angle.
   An angle coterminal to $\frac{17\pi}{6}$ is $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$
   Now, $\frac{5\pi}{6}$ is in Quadrant II. Therefore, the reference angle is $\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \boxed{\frac{\pi}{6}}$

How to Evaluate Trigonometric Functions Using Reference Angles
Note: Recall the Quadrants, A-S-T-C, SOH-CAH-TOA, and the reciprocal identities.
1. Determine if the angle is between 0 and $2\pi$, if not find an angle coterminal to this angle that does.
3. Determine the Quadrant in which the angle lies.
4. Determine whether the trig function is positive or negative in that quadrant.
5. Find the reference angle.
6. Evaluate the trig function using the sign, the reference angle and comparing to the abovementioned standard right triangles.

Example: Evaluate.

3. $\sin\left(\frac{10\pi}{3}\right)$

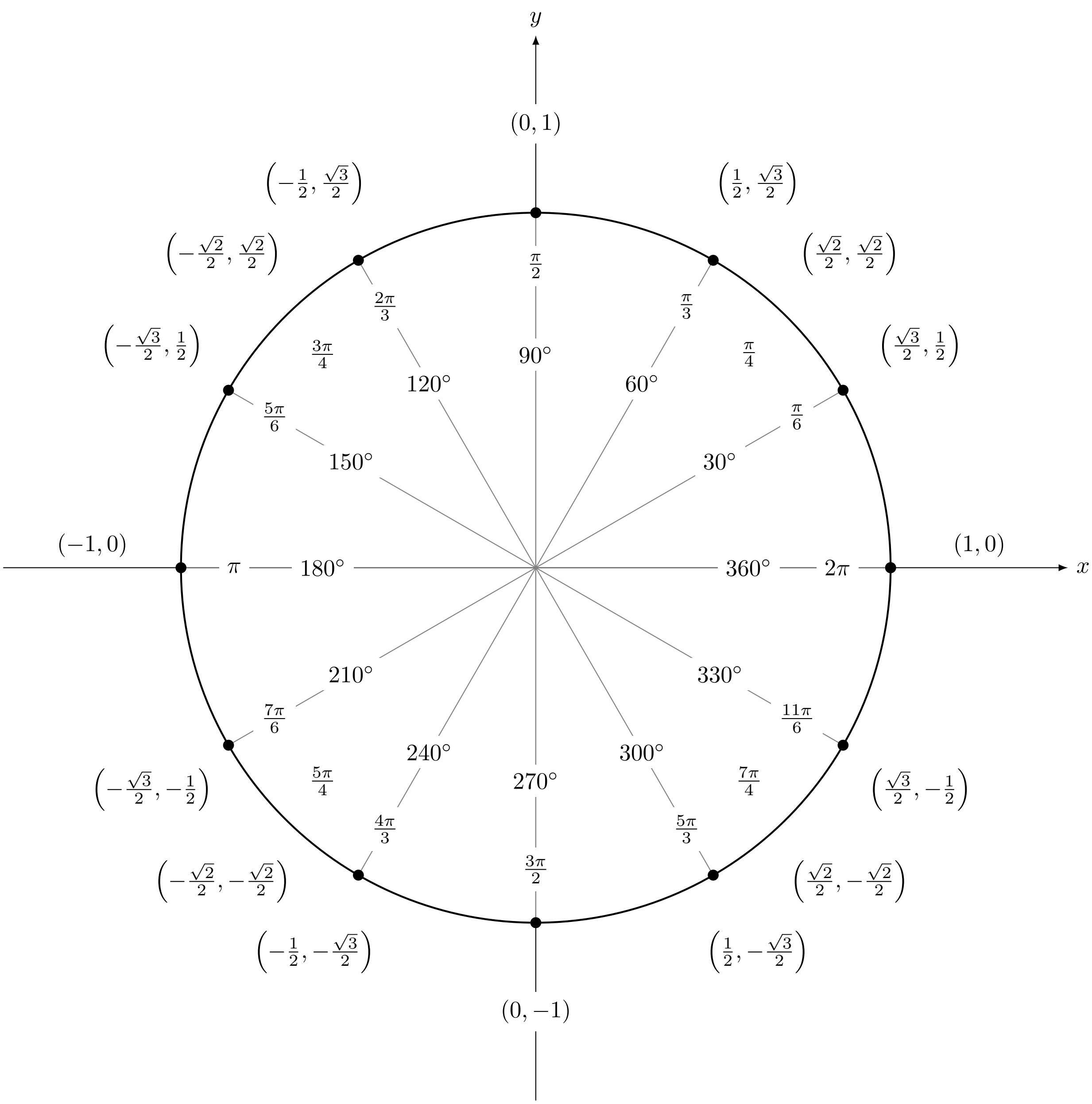
   Solution: Notice, that $\frac{10\pi}{3} > 2\pi$ , so an angle coterminal to $\frac{10\pi}{3}$ is $\frac{10\pi}{3} - 2\pi = \frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$
   $\therefore \sin\left(\frac{10\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right)$ and the angle $\frac{4\pi}{3}$ lies in Quadrant III. sine is negative in Quadrant III.

The reference angle is $\dfrac{4\pi}{3} - \pi = \dfrac{4\pi}{3} - \dfrac{3\pi}{3} = \dfrac{\pi}{3}$

$$\therefore \sin\left(\frac{10\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$$

Trigonometric Functions of Quadrantal Angles:

Recall the unit circle (shown in the image below). Along the unit circle the coordinates $(x, y) = (\cos\theta, \sin\theta)$. Using this information, your trigonometric relationships, including the reciprocal identities, you may easily evaluate trigonometric functions at quadrantal angles.




Example: Using the unit circle evaluate all six trigonometric functions (sine, cosine, tangent, secant, cosecant, and cotangent) given the angle 0 radians.

Solution:

At 0 radians, the point on the unit circle is $(1, 0) \therefore (1, 0) = (\cos 0, \sin 0)$
$\cos 0 = 1$ and $\sin 0 = 0$

$$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0 \text{ and } \cot 0 = \frac{\cos 0}{\sin 0} = \text{undefined since } \sin 0 = 0$$
$$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1 \text{ and } \csc 0 = \frac{1}{\sin 0} = \text{undefined since } \sin 0 = 0$$

## 7.2 Exercises

Given the following angle, evaluate all six trigonometric functions (sine, cosine, tangent, secant, cosecant, and cotangent). Do NOT use any electronic device!

1. $\frac{\pi}{2}$
2. $\frac{2\pi}{3}$
3. $\frac{3\pi}{4}$
4. $\frac{5\pi}{6}$
5. $\pi$
6. $\frac{7\pi}{6}$
7. $\frac{5\pi}{4}$
8. $\frac{4\pi}{3}$
9. $\frac{3\pi}{2}$
10. $\frac{5\pi}{3}$
11. $\frac{7\pi}{4}$
12. $\frac{11\pi}{6}$
13. $2\pi$
14. $\frac{13\pi}{6}$
15. $\frac{9\pi}{4}$
16. $\frac{7\pi}{3}$
17. $5\pi$
18. $\frac{5\pi}{2}$
19. $\frac{11\pi}{2}$

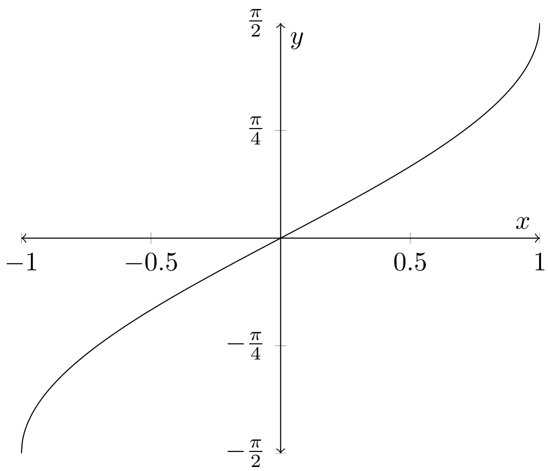
# 8 Inverse Trig Functions

Recall: Only functions that are one-to-one have inverse functions. Trigonometric functions are not one-to-one, but by restricting the domains of the trigonometric functions we can make them one-to-one and the restricted trigonometric functions will therefore have inverse functions.

## 8.1 Restricted Trig Functions & their Inverse Functions:

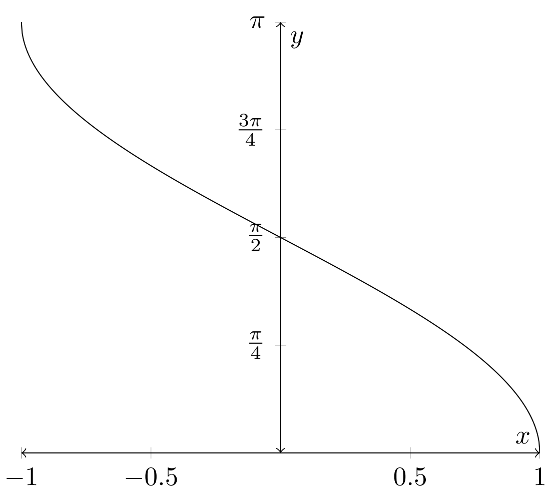
1. $y = \sin x; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

1. $y = \sin^{-1} x = \arcsin x$; where $-1 \leq x \leq 1$



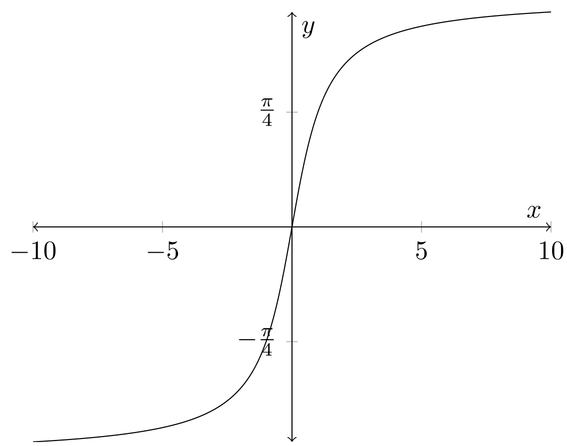

2. $y = \cos x; 0 \leq x \leq \pi$

2. $y = \cos^{-1} x = \arccos x$; where $-1 \leq x \leq 1$



3. $y = \tan x; -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

3. $y = \tan^{-1} x = \arctan x$; where $x \in \mathbb{R}$




Examples: Evaluate without using a calculator.

1. $\arcsin\left(-\dfrac{1}{2}\right)$

Solution:

$\boxed{\arcsin\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}}$ $\because \sin\left(-\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$ and the range of $y = \arcsin x$ is $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$

2. $\arccos\left(-\dfrac{\sqrt{3}}{2}\right)$

Solution:

$\boxed{\arccos\left(-\dfrac{\sqrt{3}}{2}\right) = \dfrac{5\pi}{6}}$ $\because \cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$ and the range of $y = \arccos x$ is $0 \leq x \leq \pi$

3. $\lim\limits_{x\to\infty} \arctan x$

Solution:

$\boxed{\lim\limits_{x\to\infty} \arctan x = \dfrac{\pi}{2}}$

## 8.2 Exercises

#1 - 10 Find the exact value of each expression without using a calculator.

1. $\arcsin\left(\dfrac{\sqrt{3}}{2}\right)$

2. $\cos^{-1}(-1)$

3. $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

4. $\sec^{-1}(2)$

5. $\arctan(1)$

6. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

7. $\arccos\left(-\frac{1}{2}\right)$

8. $\cot^{-1}\left(-\sqrt{3}\right)$

9. $\arcsin\left(\sin\left(\frac{7\pi}{3}\right)\right)$

10. $\tan\left(\tan^{-1}(10)\right)$

#11 - 15 Evaluate the limit.

11. $\lim_{x \to -\infty} \arctan x$

12. $\lim_{x \to 3^+} \arctan\left(\frac{1}{x-3}\right)$

13. $\lim_{x \to 3^-} \arctan\left(\frac{1}{x-3}\right)$

14. $\lim_{x \to 0^+} \tan^{-1}(\ln x)$

15. $\lim_{x \to \infty} \arcsin\left(\frac{1-x^2}{1+2x^2}\right)$

# 9 Differentiation Techniques

## 9.1 Differentiation Rules

Theorem: The Product Rule Let $u$ and $v$ be differentiable functions of $x$, then $\boxed{\frac{d}{dx}\left[uv\right] = u'v + uv'}$.

Examples: Find the derivative.

1. $y = e^x \arctan x$

   Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\right)\arctan x + e^x \frac{d}{dx}\left(\arctan x\right) = e^x \arctan x + e^x \left(\frac{1}{1+x^2}\right)$$

$$\therefore \boxed{\frac{dy}{dx} = e^x \arctan x + \frac{e^x}{1+x^2}}$$

2. $y = (\ln x)\left(4x^3 + 9x + 8\right)$

   Solution:

$$\frac{dy}{dx} = \frac{d}{dx}(\ln x)\left(4x^3+9x+8\right) + \ln x \frac{d}{dx}\left(4x^3+9x+8\right) = \frac{1}{x}\left(4x^3+9x+8\right) + (\ln x)\left(12x^2+9\right)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{4x^3+9x+8}{x} + (\ln x)\left(12x^2+9\right)}$$

Theorem: The Quotient Rule Let $u$ and $v$ be differentiable functions of $x$, then $\boxed{\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}}$.

Examples: Find $\frac{dy}{dx}$.

3. $y = \frac{5\cos x}{x^2+1}$.

   Solution:

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(5\cos x)(x^2+1) - 5\cos x \frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \frac{(-5\sin x)(x^2+1) - 5\cos x(2x)}{(x^2+1)^2}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{(-5\sin x)(x^2+1) - 10x\cos x}{(x^2+1)^2}}$$

4. $= \frac{9\tan x}{2x^5 - 3x + 1}$

   Solution:

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(9\tan x)(2x^5-3x+1) - 9\tan x \frac{d}{dx}(2x^5-3x+1)}{(2x^5-3x+1)^2} = \frac{9\sec^2 x(2x^5-3x+1) - (9\tan x)(10x^4-3)}{(2x^5-3x+1)^2}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{9\sec^2 x(2x^5-3x+1) - (9\tan x)(10x^4-3)}{(2x^5-3x+1)^2}}$$

Definition: The Chain Rule Let $f$ and $g$ be functions. For all $x$ in the domain of $g$ for which $g$ is differentiable at $x$ and $f$ differentiable at $g(x)$, the derivative of the composite function is: $\boxed{\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)}$.

Alternatively, if $y$ is a function of $u$, and $u$ is a function of $x$ then $\boxed{\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}}$.

Examples: Differentiate.

5. $y = \sqrt[4]{7x^3 - 6x + 11}$.

Solution:

$$\frac{dy}{dx} = \frac{1}{4}\left(7x^3 - 6x + 11\right)^{-3/4} \cdot \frac{d}{dx}\left(7x^3 - 6x + 11\right) = \frac{1}{4}\left(7x^3 - 6x + 11\right)^{-3/4}\left(21x^2 - 6\right)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{1}{4}\left(7x^3 - 6x + 11\right)^{-3/4}\left(21x^2 - 6\right)}$$

6. $y = \ln\left(7x^6 - 4x^5 + x^2 + 16\right)$

Solution:

$$\frac{dy}{dx} = \frac{1}{7x^6 - 4x^5 + x^2 + 16} \cdot \frac{d}{dx}\left(7x^6 - 4x^5 + x^2 + 16\right) = \frac{1}{7x^6 - 4x^5 + x^2 + 16} \cdot \left(42x^5 - 20x^4 + 2x\right)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{42x^5 - 20x^4 + 2x}{7x^6 - 4x^5 + x^2 + 16}}$$

## 9.2 Exercises

Differentiate.

1. $y = (5x + e)\sin(9x)$
2. $y = \dfrac{10x^2 - 4x + 3}{5 - 3x}$
3. $y = \sqrt[8]{x}\ln\left(5x^2 + 1\right)$
4. $y = \sqrt[3]{x^4 + 5x + 9}\ln x$
5. $y = \dfrac{e^{-2x}}{x^2 - 4}$
6. $y = \dfrac{(x+1)(x-2)}{7 - x^3}$
7. $y = -6\sec(4x)$
8. $y = 7\csc\left(x^2\right) - 2\cot(\pi x)$
9. $y = 8\left(x^4 + 5x\right)^{10}$
10. $y = 2e^{-2x^3}(4 - \ln x)^{-3}$
11. $y = \sin\left(2x^5 - 2x + 7\right)$
12. $y = \sec\left(2x^9\right)$
13. $y = \cos^6\left(\sin\left(3x^4\right)\right)$
14. $y = e^{15x^2 + 2x}$
15. $y = e^{-8x}\sin(3x)$
16. $y = \ln\left(x^4 - 3x^2 + 24\right)$
17. $y = x^{20}\ln\sqrt{x^2 + 2x + 18}$
18. $y = \dfrac{4}{e^x + e^{-x}}$
19. $y = \sin^{12}\left(\tan\left(3x^5\right)\right)$
20. $y = 4\sin(\ln x)$
21. $y = 5\cot\left(e^x\right)$
22. $y = -3\csc(4x)$
23. $y = e^{6x^2}\tan(8x)$
24. $y = \ln\left(7x^3 + 5x - 19\right)$
25. $y = \ln(\cos(\pi x))$
26. $y = \left(\ln\left(4x^2\right)\right)^{12}$
27. $y = \cos(\sin x)$
28. $y = \sin^{17}(\cos(7x))$
29. $y = \cos^{16}\left(\tan\left(3x^3\right)\right)$
30. $y = \ln\left(8x^4 - 11x^2 + 3\right)$
31. $y = \sqrt[6]{2x^3 - 3x + 1}$
32. $y = e^{6x^2 + 5x - 7}$
33. $y = \sqrt{x}\ln\left(11 + 9x^2\right)$
34. $y = 9x^4\csc(7x)$
35. $y = \cos^4(7x)$
36. $y = \tan\left(\sqrt[4]{x}\right)$
37. $y = \sin^{10}\left(\sin\left(2x^5\right)\right)$
38. $y = e^{-x}\ln 2 + x^5$

# 10 L'Hôpital's Rule

## 10.1 Theorems & Methods

Theorem: L'Hôpital's Rule 0/0 case Suppose $f$ and $g$ are differentiable functions over an open interval containing $c$ except possibly at $c$. If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$ then $\boxed{\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}}$ assuming that the limit on right exists or is $\pm\infty$. The result also holds for one-sided limits or if $c = \pm\infty$.

Theorem: L'Hôpital's Rule $\infty/\infty$ case Suppose $f$ and $g$ are differentiable functions over an open interval containing $c$ except possibly at $c$. If $\lim_{x\to c} f(x) = \pm\infty$ and $\lim_{x\to c} g(x) = \pm\infty$ then $\boxed{\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}}$

Examples: Determine the limit.

1. $\lim_{x\to 1} \frac{2x-2}{x^2-1}$

   Solution:
   Notice, this is a 0/0 case. While you can factor and simplify, we can also apply L'Hôpital's Rule.

   $$\lim_{x\to 1} \frac{2x-2}{x^2-1} \overset{LH}{=} \lim_{x\to 1} \frac{2}{2x} = \frac{2}{2(1)} = \boxed{1}$$

2. $\lim_{x\to\infty} \frac{x^2-1}{2x^2+1}$

   Solution:
   Notice, this is a $\infty/\infty$ case. $\lim_{x\to\infty} \frac{x^2-1}{2x^2+1} \overset{LH}{=} \lim_{x\to\infty} \frac{2x}{4x} = \lim_{x\to\infty} \frac{1}{2} = \boxed{\frac{1}{2}}$

3. $\lim_{x\to 1} \frac{\ln x}{x-1}$

   Solution:
   Notice, this is a 0/0 case. $\lim_{x\to 1} \frac{\ln x}{x-1} \overset{LH}{==} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \lim_{x\to 1} \frac{1}{x} = \frac{1}{1} = \boxed{1}$

4. $\lim_{x\to\infty} \frac{e^x}{x^2}$

   Solution: Notice, this is a $\infty/\infty$ case. $\lim_{x\to\infty} \frac{e^x}{x^2} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{LH}{==} \lim_{x\to\infty} \frac{e^x}{2} = \boxed{\infty}$

Indeterminate Products & Indeterminate Differences: $0\cdot\infty, \infty-\infty$
In order to determine if a limit exists or does not exists, to handle these indeterminate forms, the strategy is to re-write the expression to attain one of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and then apply L'Hôpital's Rule to evaluate the limit.

Examples: Determine the limit.

5. $\lim_{x\to 0^+} x\ln x$

   Solution: Notice, this is a $0\cdot -\infty$ case, so first, rewrite $f(x) = x\ln x as \frac{\ln x}{x^{-1}}$ $\therefore$, $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1}}$

   Now, this is a $-\infty/\infty$ case. $\lim_{x\to 0^+} \frac{\ln x}{x^{-1}} \overset{LH}{==} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-x^{-2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = \lim_{x\to 0^+} -x = \boxed{0}$

6. $\lim_{x\to(\pi/2)^-} (\sec x - \tan x)$

Solution:

Notice, this is a $\infty - \infty$ case. $\lim_{x\to(\pi/2)^-} (\sec x - \tan x) = \lim_{x\to(\pi/2)^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) = \lim_{x\to(\pi/2)^-} \frac{1-\sin x}{\cos x}$

Now, this is a 0/0 case. $\lim_{x\to(\pi/2)^-} \frac{1-\sin x}{\cos x} \overset{LH}{=} \lim_{x\to(\pi/2)^-} \frac{-\cos x}{-\sin x} = \frac{0}{1} = \boxed{0}$

Indeterminate Powers: $1^\infty, 0^0, \infty^0$.

Assume that the limit exists and set it equal to y, then take the natural logarithm of both sides. Since the natural logarithm is a continuous function, you may interchange the limit and the natural logarithm. Rewrite the expression to achieve an indeterminate form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$ so that you may apply L'Hôpital's Rule to evaluate the limit. Then solve for $y$.

Example: Determine the limit.

7. $\lim_{x\to 0^+} (1+\sin(4x))^{\cot x}$

Solution:

Notice this is a $1^\infty$ case, so $y = \lim_{x\to 0^+} (1+\sin(4x))^{\cot x}$ $\therefore \ln y = \ln\left(\lim_{x\to 0^+} (1+\sin(4x))^{\cot x}\right) = \lim_{x\to 0^+} \ln\left[(1+\sin(4x))^{\cot x}\right]$

$\therefore \ln y = \lim_{x\to 0^+} (\cot x)\ln\left[(1+\sin(4x))\right]$ and this is a $0\cdot\infty$ case.

$\therefore \ln y = \lim_{x\to 0^+} \frac{\ln\left[(1+\sin(4x))\right]}{\tan x}$ and this is a 0/0 case.

$\therefore \ln y = \lim_{x\to 0^+} \frac{\ln\left[(1+\sin(4x))\right]}{\tan x} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{4\cos(4x)}{1+\sin(4x)}}{\sec^2 x} = \lim_{x\to 0^+} \frac{4\cos(4x)}{\sec^2 x(1+\sin(4x))} = \frac{4(1)}{1(1+0)} = 4$

$\therefore \ln y = 4 \Rightarrow y = e^4 \therefore \boxed{\lim_{x\to 0^+} (1+\sin(4x))^{\cot x} = e^4}$.

## 10.2 Exercises

Find the limit.

1. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$
2. $\lim_{x\to 0} \frac{\sin(4x)}{5x}$
3. $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$
4. $\lim_{x\to\infty} \frac{\sqrt{x}}{3x^2}$
5. $\lim_{x\to -5} \frac{x^2+10x+25}{x+5}$
6. $\lim_{x\to -4} \frac{x^2-16}{3x^2-3x-60}$
7. $\lim_{x\to\infty} \frac{3x^4-4x}{2x^2-5x^4}$
8. $\lim_{x\to\infty} \frac{5x^8+18}{6x^8+7x^3+1}$
9. $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$
10. $\lim_{x\to\infty} x^3 e^{-x^2}$
11. $\lim_{x\to\infty} \frac{e^x}{x^2}$
12. $\lim_{x\to\infty} \frac{\sqrt{16x+7}}{\sqrt{x+5}}$
13. $\lim_{x\to 0} \frac{5^x-3^x}{x}$
14. $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$

15 $\lim_{x\to 0^+} \sin x \ln x$

16. $\lim_{x\to 0^+} \frac{\ln x}{\cot x}$
17. $\lim_{x\to\infty} x^{x^2}$
18. $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx}$

# 11 Integration by Substitution

## 11.1 Method: $u$-sub

Method of Integration by Substitution: $\boxed{\int f'(g(x))g'(x)dx = \int f'(u)du;\ \text{where } u = g(x)}$.

1. Let $u$ be a function of $x$; usually part of the integrand.
2. Find the differential.
3. Convert the integral to a $u$-variable form.
4. Integrate.
5. Substitute back so the anitiderivative is a function of $x$.

Examples: Integrate.

1. $\int x^2\sqrt{x^3+5}dx$

   Solution: Let $u = x^3 + 5 \therefore du = 3x^2dx$ so $\frac{1}{3}du = x^2dx$

   $$\int x^2\sqrt{x^3+5}dx = \int \frac{1}{3}u^{1/2}du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{9}\left(x^3+5\right)^{3/2} + C}$$

2. $\int \frac{1}{5x+6}dx$

   Solution: Let $u = 5x + 6 \therefore du = 5dx$ so $\frac{1}{5}du = dx$

   $$\int \frac{1}{5x+6}dx = \int \frac{1}{5}\cdot\frac{1}{u}du = \frac{1}{5}\ln|u| + C = \boxed{\frac{1}{5}\ln|5x+6|}$$

Change of Variables for Definite Integrals $\boxed{\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du;\ \text{where } u = g(x)}$.

Examples: Evaluate.

3. $\int_1^e \frac{\ln x}{x}dx$

   Solution: Let $u = \ln x \therefore du = \frac{1}{x}dx$ and $x = 1 \Rightarrow u = \ln 1 = 0$ and $x = e \Rightarrow u = \ln e = 1$

   $$\int_1^e \frac{\ln x}{x}dx = \int_0^1 udu = \frac{1}{2}u^2\Big|_0^1 = \frac{1}{2}(1-0) = \boxed{1}$$

4. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}dx$

   Solution: Let $u = 1 + 2x^2 \therefore du = 4xdx$ so $\frac{1}{4}du = xdx$

   and $x = 0 \Rightarrow u = 1 + 2(0) = 1$ and $x = 2 \Rightarrow u = 1 + 2(4) = 9$

   $$\int_0^2 \frac{x}{\sqrt{1+2x^2}}dx = \int_1^9 \frac{1}{4}\cdot\frac{1}{\sqrt{u}}du = \int_1^9 \frac{1}{4}u^{-1/2}du = \frac{1}{4}\left(2u^{1/2}\right)\Big|_1^9 = \frac{1}{2}u^{1/2}\Big|_1^9 = \frac{1}{2}\left(\sqrt{9}-\sqrt{1}\right) = \frac{1}{2}(2) = \boxed{1}$$

### 11.2 Exercises

# 1-13 Find the indefinite integral.

1. $\int 5x\sqrt[3]{x^2+1}dx$

2. $\int \frac{\cos\sqrt{3x+2}}{\sqrt{3x+2}}dx$

3. $\int \frac{\sec^2(\ln x)}{x}dx$

4. $\int \frac{3x+7}{1+x^2}dx$

5. $\int \frac{e^{2x}}{1+e^{4x}}dx$

6. $\int 4x^3\csc(5x^4)\cot(5x^4)dx$

7. $\int \frac{x^3+2}{x^4+8x}dx$

8. $\int x\cos(x^2+1)+\frac{1}{x^2+1}dx$

9. $\int \frac{1}{8x+9}dx$

10. $\int \frac{1}{2+8x^2}dx$

11. $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx$

12. $\int e^{x+e^x}dx$

13. $\int \left(4\cos^3(x)+6\cos^2(x)-8\right)\sin x dx$

# 14-15 Evaluate.

14. $\int_0^{\pi/2}\cos x\sin(\sin x)dx$

15. $\int_1^2 x(x^2-1)^5dx$

# 12 Parametric Equations

## 12.1 Definitions & Guidelines

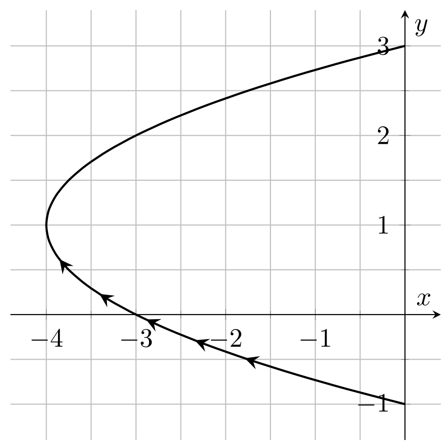
Definition: Parametric equations are a set of equations, say $x = f(t), y = g(t)$, as functions of an independent third variable.

Example:

1. Sketch the parametric curve for the set of parametric equations $x = t^2 - 4t, y = t - 1$. Be sure to include orientation of the curve. Then find an explicit relationship between $x$ and $y$ by eliminating the parameter.

Solution:

| $t$ | $x$ | $y$ |
|---|---|---|
| $-1$ | 5 | $-2$ |
| 0 | 0 | $-1$ |
| 1 | $-3$ | 0 |
| 2 | $-4$ | 1 |
| 3 | $-3$ | 2 |
| 4 | 0 | 3 |




$y = t - 1 \Rightarrow t = y - 1 \therefore$ by substituting this into the equation for $x$ the explicit relationship is

$x = (y+1)^2 - 4(t+1) \therefore \boxed{x = (y+1)(y-3)}$

Guidelines: How to determine a set of parametric equations for a given curve:

1. ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, counter-clockwise: $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2\pi$

2. circle $x^2 + y^2 = r^2$, counter-clockwise: $x = r\cos t,\ y = r\sin t,\ 0 \le t \le 2\pi$

3. line segment from $(x_0, y_0, z_0)$ to $(x_1, y_1, z_1)$, $\vec{r}(t) = (1-t) < x_0, y_0, z_0 > +t < x_1, y_1, z_1 >,\ 0 \le t \le 1$

4. $y = f(x),\ x = t,\ y = f(t)$

Example:

2. Find a set of parametric equations that represent traveling clockwise around a circle centered at the origin with radius 3, twice.

Solution:

A circle centered at the origin with a radius of 3 has an equation $x^2 + y^2 = 9$

Since it is indicated that the orientation is clockwise the form of the parametric equations are

$x = r\sin t, y = r\cos t, 0 \le t < 2\pi$

Also, it indicates that the the circle is being traveled twice, which indicates the frequency of the trig functions is 2.

$\therefore \boxed{x = 3\sin(2t), y = 3\cos(2t), 0 \le t < 2\pi}$

Note: One may check by using this set of parametric equations and determining the explicit relationship to ensure that it is $x^2 + y^2 = 9$ .

## 12.2 Exercises

#1-2 Sketch the parametric curve for the set of parametric equations. Be sure to include orientation of the curve. Then find an explicit relationship between $x$ and $y$ by eliminating the parameter.

1. $x = \cos 2\theta,\ y = \sin 2\theta;\ 0 \le \theta \le 2\pi$

2. $x = \sqrt{t},\ y = t - 5$

# 3-6 Find a set of parametric equations that satisfy the given information.

3. circle with center $(2, 3)$ and radius 4

4. ellipse with vertices $(\pm 10, 0)$; foci $(\pm 8, 0)$

5. $y = 4x + 1$, $t = -1$ at the point $(-2, -7)$.

6. $y = x^2$, $t = 4$ at the point $(4, 16)$.

# 13 Polar

## 13.1 Conversion Formulas & Graphing

Polar to Rectangular Coordinate Conversion: $\boxed{x = r\cos\theta \text{ and } y = r\sin\theta}$

Examples: Convert from polar coordinates to rectangular coordinates.

1. $\left(5, \frac{2\pi}{3}\right)$

   Solution:

   $$x = r\cos\theta \therefore x = 5\cos\left(\frac{2\pi}{3}\right) = 5\left(-\frac{1}{2}\right) = -\frac{5}{2}$$
   $$y = r\sin\theta \therefore x = 5\sin\left(\frac{2\pi}{3}\right) = 5\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{2}$$

   $\therefore \boxed{\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)}$

2. $\left(-3, \frac{3\pi}{2}\right)$

   Solution:

   $$x = r\cos\theta \therefore x = -3\cos\left(\frac{3\pi}{2}\right) = -3(0) = 0$$
   $$y = r\sin\theta \therefore x = -3\sin\left(\frac{3\pi}{2}\right) = -3(-1) = 3$$

   $\boxed{(0, 3)}$

Rectangular to Polar Coordinate Conversion: $\boxed{\tan\theta = \frac{y}{x} \text{ and } r^2 = x^2 + y^2}$

Examples: Convert from rectangular coordinates to polar coordinates.

3. $\left(\frac{-7\sqrt{3}}{2}, -\frac{7}{2}\right)$

   Solution:

   $$r^2 = x^2 + y^2 \therefore r^2 = \frac{49 \cdot 3}{4} + \frac{49}{4} = \frac{4(49)}{4} = 49, \text{ so } r = \pm 7$$

   First determine the reference angle: $\theta_{ref} = \arctan\left|\frac{-7/2}{-7\sqrt{3}/2}\right| = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$.

   Notice that the point is in Quadrant III. Therefore, with $r = 7, \theta = \frac{7\pi}{6}$ and with $r = -7, \theta = \frac{\pi}{6}$

   $\therefore \boxed{\left(7, \frac{7\pi}{6}\right) = \left(-7, \frac{\pi}{6}\right)}$ Please note that there are other equivalent possibilities as $\theta$ could also be negative.

4. $\left(-1, \sqrt{3}\right)$

Solution:

$r^2 = x^2 + y^2 \therefore r^2 = 1 + 3 = 4,\ \text{so } r = \pm 2$

First determine the reference angle: $\theta_{ref} = \arctan\left|\frac{\sqrt{3}}{-1}\right| = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$.

Notice that the point is in Quadrant II. Therefore, with $r = 2, \theta = \frac{2\pi}{3}$ and with $r = -2, \theta = \frac{5\pi}{3}$

$\therefore \boxed{\left(2, \frac{2\pi}{3}\right) = \left(-2, \frac{5\pi}{3}\right)}$ Please note that there are other equivalent possibilities as $\theta$ could also be negative.

Methods for graphing polar equations:
1. Convert to rectangular equation
2. Make a table of values
3. Recognize a special polar graph (see information on special types below)
4. Use technology (you will not be allowed to use technology on your exams!).

Special Polar Graphs:

A. Limaçons – Form: $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$ for $a > 0, b > 0$.

1. For $a < b$ we have a limaçons with an inner loop
2. For $a = b$ we have a cardiod
3. For $1 < \frac{a}{b} < 2$ we have a dimpled limaçon
4. For $\frac{a}{b} \geq 2$ we have a convex limaçon

B. Roses – Form: $r = a\sin(n\theta)$ or $r = a\cos(n\theta)$, $n \geq 2$

1. Petal length = $|a|$
2. the number of petal is $n$ if $n$ is odd and $2n$ if n is even.
3. Tip of first petal when form has cosine is always at $\theta = 0$ (since this is when $\cos(n\theta) = 1$).
4. Tip of first petal when form has sine is $\frac{90^\circ}{n} = \frac{\pi}{2n}$ (since this is when $\sin(n\theta) = 1$)

C. Circles – Form $r = a\cos\theta$ or $r = a\sin\theta$

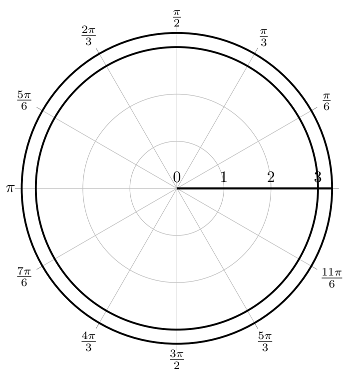
D. Lemniscates – Form $r^2 = \pm a^2\sin(2\theta)$ or $r^2 = \pm a^2\cos(2\theta)$

1. Graph looks like a figure 8.
2. For sine, it will be symmetric about the pole, where tips will be at either $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ or $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ depending on the sign being positive or negative; respectively.
3. For cosine, it will symmetric about the polar axis, where tips will be at either 0 and $\pi$ or $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ depending on the sign being positive or negative; respectively.

Examples: Sketch the graph of the polar curve.

5. $r = 3$

Solution: Notice, that $r$ is fixed, but $\theta$ can be any angle.

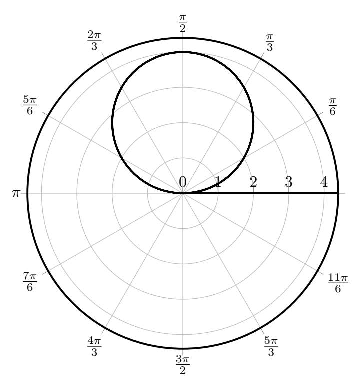



Notice, also that the corresponding rectangular equation would be: $r = 3 \Rightarrow r^2 = 9 \Rightarrow x^2 + y^2 = 9$

6. $r = 4 \sin \theta$

Solution: This is the form of a circle.

| $r$ | $\theta$ |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{6}$ | $2$ |
| $\frac{\pi}{2}$ | $4$ |
| $\frac{5\pi}{6}$ | $2$ |
| $\pi$ | $0$ |
| $\frac{7\pi}{6}$ | $-2$ |
| $\frac{3\pi}{2}$ | $-4$ |
| $\frac{11\pi}{6}$ | $-2$ |
| $2\pi$ | $0$ |




Notice, that over the interval $[0, 2\pi]$ the circle is traveled twice. In addition, the corresponding rectangular equation would be:

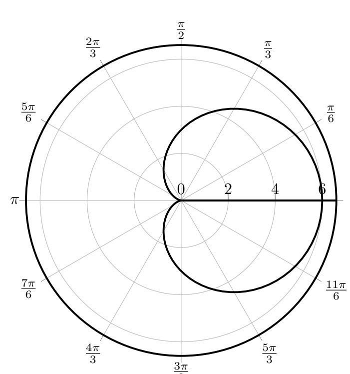
$$r = 4 \sin \theta \Rightarrow r^2 = 4r \sin \theta$$
$$\Rightarrow x^2 + y^2 = 4x \Rightarrow x^2 - 4x + y^2 = 0$$
$$\therefore (x-2)^2 + y^2 = 4$$

Notice, that this equation corresponds to the plot; a circle with center $(2, 0)$ and radius 2.

7. $r = 3 + 3 \cos \theta$

Solution: Notice this is the graph of a limaçon with $a = b$ so it is a cardiod.

| $r$ | $\theta$ |
|---|---|
| $0$ | $6$ |
| $\frac{\pi}{3}$ | $\frac{9}{2}$ |
| $\frac{\pi}{2}$ | $3$ |
| $\frac{2\pi}{3}$ | $\frac{3}{2}$ |
| $\pi$ | $0$ |
| $\frac{4\pi}{3}$ | $\frac{3}{2}$ |
| $\frac{3\pi}{2}$ | $3$ |
| $\frac{5\pi}{3}$ | $\frac{9}{2}$ |
| $2\pi$ | $6$ |



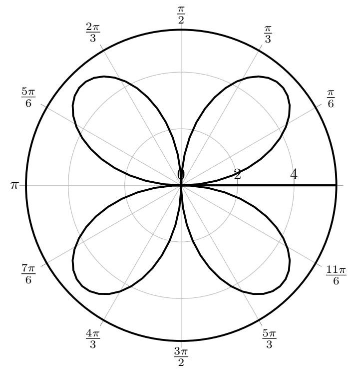
8. $r = 5\sin(2\theta)$

<u>Solution:</u> Notice, this is the equation of a Rose.

Identify from the equation: $a = 5$ and $n = 2$
Petal length is $a = 5$
$n$ is even so the number of petals is $2n \therefore 4$ petals.
Spacing between petals is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$
Location of the tip of the first petal is when:

$$2\theta = \frac{\pi}{2} \therefore \theta = \frac{\pi}{4}$$

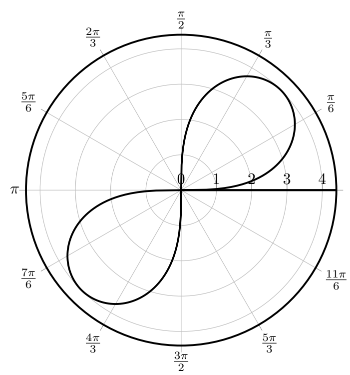
Locations of Petal Tips: $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$




9. $r^2 = 16\sin(2\theta)$

<u>Solution:</u> This is the equation of a lemniscate.

Since it is sine, it will symmetric about the pole. Notice, the sign is positive, therefore, the graph looks like a figure 8 that is symmetric about the pole where the tips will be at $\dfrac{\pi}{4}$ and $\dfrac{3\pi}{4}$.




## 13.2 Exercises

# 1-2 Convert from polar coordinates to rectangular coordinates.

1. $\left(-6, \dfrac{4\pi}{3}\right)$

2. $\left(8, -\dfrac{5\pi}{6}\right)$

# 3-4 Convert from rectangular coordinates to polar coordinates. Find two representations for the point in polar coordinates where $0 \le \theta < 2\pi$.

3. $(-5, 5)$

4. $\left(-\sqrt{3}, -1\right)$

# 5-8 Sketch (by hand) the graph of the polar equation.

5. $r = 6\cos\theta$

6. $r = 3 + 3\sin\theta$

7. $r = 4\cos(2\theta)$

8. $r = 5\sin(4\theta)$

# A Answers to Exercises

## A.1 Section 4.2

1. $\dfrac{x^2(6x-1)}{2(5x^3-4)}$

2. $\dfrac{x-5}{2(x+4)}$

## A.2 Section 5.2

1. $y=(x+3)^2+2$

2. $y=-\left(x-\dfrac{5}{2}\right)^2-\dfrac{103}{4}$

3. $y=2(x+2)^2+15$

4. $y=-3(x-1)^2-11$

5. $y=2\left(x+1\right)^2-\dfrac{57}{8}$

## A.3 Section 6.2

1. 0

2. 1

3. 2

4. $-2$

5. 6

6. $10x^3+7$

7. $3x$

8. $\sqrt{x+1}$

9. $3\log x+\dfrac{1}{4}\log\left(5x^2+9\right)$

10. $6\log_5\left(x^2+1\right)+7\log_5\left(1-x^3\right)-9\log_5\left(x^2+x+1\right)$

11. $\dfrac{1}{2}\ln\left(x^4+9\right)-5\ln\left(x^3+7x\right)-\dfrac{1}{3}\ln\left(2x+1\right)$

12. $\dfrac{1}{5}\Big[4\ln\left(5x+9\right)+3\ln\left(x-8\right)-2\ln\left(7x+11\right)\Big]$

13. $\ln\left(\dfrac{(x+8)^4(x^5+9)^2}{\sqrt{3x-7}}\right)$

14. $\ln\left(\dfrac{(2x+11)^2(x^7+1)^2}{(x-9)^5}\right)$

15. $\log_2\left(\dfrac{\sqrt[3]{5-x}}{(5x+8)^4(x^3+2)^3}\right)$

16. $\log\left(\dfrac{\sqrt[3]{(9x+1)^2}}{(x+2)^5(x+3)^5}\right)$

## A.4 Section 7.2

1. $\sin\left(\dfrac{\pi}{2}\right)=1, \cos\left(\dfrac{\pi}{2}\right)=0, \tan\left(\dfrac{\pi}{2}\right)=$ undefined, $\csc\left(\dfrac{\pi}{2}\right)=1, \sec\left(\dfrac{\pi}{2}\right)=$ undefined, $\cot\left(\dfrac{\pi}{2}\right)=0$

2. $\sin\left(\dfrac{2\pi}{3}\right)=\dfrac{\sqrt{3}}{2}, \cos\left(\dfrac{2\pi}{3}\right)=-\dfrac{1}{2}, \tan\left(\dfrac{2\pi}{3}\right)=-\sqrt{3}, \csc\left(\dfrac{2\pi}{3}\right)=\dfrac{2}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}, \sec\left(\dfrac{2\pi}{3}\right)=-2, \cot\left(\dfrac{2\pi}{3}\right)=-\dfrac{1}{\sqrt{3}}=-\dfrac{\sqrt{3}}{3}$

3. $\sin\left(\dfrac{3\pi}{4}\right)=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}, \cos\left(\dfrac{3\pi}{4}\right)=-\dfrac{1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}, \tan\left(\dfrac{3\pi}{4}\right)=-1, \csc\left(\dfrac{3\pi}{4}\right)=\sqrt{2}, \sec\left(\dfrac{3\pi}{4}\right)=-\sqrt{2}, \cot\left(\dfrac{3\pi}{4}\right)=-1$

4. $\sin\left(\dfrac{5\pi}{6}\right)=\dfrac{1}{2}, \cos\left(\dfrac{5\pi}{6}\right)=-\dfrac{\sqrt{3}}{2}, \tan\left(\dfrac{5\pi}{6}\right)=-\dfrac{1}{\sqrt{3}}=-\dfrac{\sqrt{3}}{3}, \csc\left(\dfrac{5\pi}{6}\right)=2, \sec\left(\dfrac{5\pi}{6}\right)=-\dfrac{\sqrt{3}}{2}, \cot\left(\dfrac{5\pi}{6}\right)=-\sqrt{3}$

5. $\sin\pi=0, \cos\pi=-1, \tan\pi=0, \csc\pi=$ undefined, $\sec\pi=-1, \cot\pi=$ undefined

6. $\sin\left(\dfrac{7\pi}{6}\right)=-\dfrac{1}{2}, \cos\left(\dfrac{7\pi}{6}\right)=-\dfrac{\sqrt{3}}{2}, \tan\left(\dfrac{7\pi}{6}\right)=\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}, \csc\left(\dfrac{7\pi}{6}\right)=-2, \sec\left(\dfrac{7\pi}{6}\right)=-\dfrac{\sqrt{3}}{2}, \cot\left(\dfrac{7\pi}{6}\right)=\sqrt{3}$

7. $\sin\left(\dfrac{5\pi}{4}\right)=-\dfrac{1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}, \cos\left(\dfrac{5\pi}{4}\right)=-\dfrac{1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}, \tan\left(\dfrac{5\pi}{4}\right)=1, \csc\left(\dfrac{5\pi}{4}\right)=-\sqrt{2}, \sec\left(\dfrac{5\pi}{4}\right)=-\sqrt{2}, \cot\left(\dfrac{5\pi}{4}\right)=1$

8. $\sin\left(\dfrac{4\pi}{3}\right)=-\dfrac{\sqrt{3}}{2}, \cos\left(\dfrac{4\pi}{3}\right)=-\dfrac{1}{2}, \tan\left(\dfrac{4\pi}{3}\right)=\sqrt{3}, \csc\left(\dfrac{4\pi}{3}\right)=-\dfrac{2}{\sqrt{3}}=-\dfrac{2\sqrt{3}}{3}, \sec\left(\dfrac{4\pi}{3}\right)=-2, \cot\left(\dfrac{4\pi}{3}\right)=\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$

9. $\sin\left(\frac{3\pi}{2}\right) = -1, \cos\left(\frac{3\pi}{2}\right) = 0, \tan\left(\frac{3\pi}{2}\right) =$ undefined, $\csc\left(\frac{3\pi}{2}\right) = -1, \sec\left(\frac{3\pi}{2}\right) =$ undefined, $\cot\left(\frac{3\pi}{2}\right) = 0$

10. $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}, \tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}, \csc\left(\frac{5\pi}{3}\right) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}, \sec\left(\frac{5\pi}{3}\right) = 2, \cot\left(\frac{5\pi}{3}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

11. $\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \tan\left(\frac{7\pi}{4}\right) = -1, \csc\left(\frac{7\pi}{4}\right) = -\sqrt{2}, \sec\left(\frac{7\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{7\pi}{4}\right) = -1$

12. $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}, \cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}, \tan\left(\frac{11\pi}{6}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}, \csc\left(\frac{11\pi}{6}\right) = -2, \sec\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}, \cot\left(\frac{11\pi}{6}\right) = -\sqrt{3}$

13. $\sin 2\pi = 0, \cos 2\pi = 1, \tan 2\pi = 0, \csc 2\pi =$ undefined, $\sec 2\pi = 1, \cot 2\pi =$ undefined

14. coterminal to $\frac{13\pi}{6}$ is $\frac{\pi}{6}, \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}, \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}, \csc\left(\frac{\pi}{6}\right) = 2, \sec\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

15. coterminal to $\frac{9\pi}{4}$ is $\frac{\pi}{4}, \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \tan\left(\frac{\pi}{4}\right) = 1, \csc\left(\frac{\pi}{4}\right) = \sqrt{2}, \sec\left(\frac{\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{\pi}{4}\right) = 1$

16. coterminal to $\frac{7\pi}{3}$ is $\frac{\pi}{3}, \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \tan\left(\frac{\pi}{3}\right) = \sqrt{3}, \csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}, \sec\left(\frac{\pi}{3}\right) = 2, \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

17. coterminal to $5\pi is\pi, \sin\pi = 0, \cos\pi = -1, \tan\pi = 0, \csc\pi =$ undefined, $\sec\pi = -1, \cot\pi =$ undefined

18. coterminal to $\frac{5\pi}{2}$ is $\frac{\pi}{2}, \sin\left(\frac{\pi}{2}\right) = 1, \cos\left(\frac{\pi}{2}\right) = 0, \tan\left(\frac{\pi}{2}\right) =$ undefined, $\csc\left(\frac{\pi}{2}\right) = 1, \sec\left(\frac{\pi}{2}\right) =$ undefined, $\cot\left(\frac{\pi}{2}\right) = 0$

19. coterminal to $\frac{11\pi}{2}$ is $\frac{3\pi}{2}, \sin\left(\frac{3\pi}{2}\right) = -1, \cos\left(\frac{3\pi}{2}\right) = 0, \tan\left(\frac{3\pi}{2}\right) =$ undefined, $\csc\left(\frac{3\pi}{2}\right) = -1, \sec\left(\frac{3\pi}{2}\right) =$ undefined, $\cot\left(\frac{3\pi}{2}\right) = 0$

## A.5 Section 8.2

1. $\frac{\pi}{3}$
2. $\pi$
3. $\frac{\pi}{6}$
4. $\frac{\pi}{3}$
5. $\frac{\pi}{4}$
6. $-\frac{\pi}{4}$
7. $\frac{2\pi}{3}$
8. $-\frac{\pi}{6}$
9. $\frac{7\pi}{3}$
10. 10
11. $-\frac{\pi}{2}$
12. $\frac{\pi}{2}$
13. $-\frac{\pi}{2}$
14. $-\frac{\pi}{2}$
15. $-\frac{\pi}{6}$

## A.6 Section 9.2

1. $y' = 5\sin(9x) + 9(5x+e)\cos(9x)$
2. $y' = \dfrac{-30x^2+100x-11}{(5-3x)^2}$
3. $y' = \frac{1}{8}x^{-7/8}\ln(5x^2+1) + \dfrac{10x^{9/8}}{5x^2+1}$
4. $y' = \frac{1}{3}(x^4+5x+9)^{-2/3}(4x^3+5)\ln x + \dfrac{\sqrt[3]{x^4+5x+9}}{x}$
5. $y' = \dfrac{-2e^{-2x}(x^2+x-4)}{(x^2-4)^2}$
6. $y' = \dfrac{x^4-2x^3-6x^2+14x-7}{(7-x^3)^2}$

7. $y' = -24\sec(4x)\tan(4x)$

8. $y' = -14x\csc(x^2)\cot(x^2) + 2\pi\csc^2(\pi x)$

9. $y' = 80(x^4+5x)^9(4x^3+5)$

10. $y' = -4x^2e^{-2x^3}(4-\ln x)^{-3} + \dfrac{6e^{-2x^3}(4-\ln x)^{-4}}{x}$

11. $y' = (10x^4-2)\cos(2x^5-2x+7)$

12. $y' = 18x^8\sec(2x^9)\tan(2x^9)$

13. $y' = -72x^3\cos^5\big(\sin(3x^4)\big)\sin\big(\sin(3x^4)\big)\cos(3x^4)$

14. $y' = (30x+2)e^{15x^2+2x}$

15. $y' = -8e^{-8x}\sin(3x) + 3e^{-8x}\cos(3x)$

16. $y' = \dfrac{4x^3-6x}{x^4-3x^2+24}$

17. $y' = 10x^{19}\ln(x^2+2x+18) + \dfrac{x^{20}(x+1)}{x^2+2x+18}$

18. $y' = \dfrac{-4(e^x-e^{-x})}{(e^x+e^{-x})^2}$

19. $y' = 180x^4\sin^{11}(\tan(3x^5))\cos(\tan(3x^5))\sec^2(3x^5)$

20. $y' = \dfrac{4\cos(\ln x)}{x}$

21. $y' = -5e^x\csc^2(e^x)$

22. $y' = 12\csc(4x)\cot(4x)$

23. $y' = 12xe^{6x^2}\tan(8x) + 8e^{6x^2}\sec^2(8x)$

24. $y' = \dfrac{21x^2+5}{7x^3+5x+19}$

25. $y' = -\pi\tan(\pi x)$

26. $y' = \dfrac{24\Big[\ln(4x^2)\Big]^{11}}{x}$

27. $y' = -\sin(\sin x)\cos x$

28. $y' = -119\sin^{16}(\cos(7x))\cos(\cos(7x))\sin(7x)$

29. $y' = -144x^2\cos^{15}(\tan(3x^3)\sin(\tan(3x^3))\sec^2(3x^3)$

30. $y' = \dfrac{32x^3-22x}{8x^4-11x^2+3}$

31. $y' = \dfrac{1}{2}(2x^3-3x+1)^{-5/6}(2x^2-1)$

32. $y' = (12x+5)e^{6x^2+5x-7}$

33. $y' = \dfrac{\ln(11+9x^2)}{2\sqrt{x}} + \dfrac{18x^{3/2}}{11+9x^2}$

34. $y' = 36x^3\csc(7x) - 63x^4\csc(7x)\cot(7x)$

35. $y' = -28\cos^3(7x)\sin(7x)$

36. $y' = \dfrac{1}{4}x^{-3/4}\sec^2(\sqrt[4]{x})$

37. $y' = 100x^4\sin^9(\sin(2x^5))\cos(\sin(2x^5))\cos(2x^5)$

38. $y' = -e^{-x}\ln 2 + 5x^4$

## A.7 Section 10.2

1. 5

2. $\frac{4}{5}$

3. 0

4. 0

5. 0

6. $\frac{8}{27}$

7. $-\frac{3}{5}$

8. $\frac{5}{6}$

9. 0

10. 0

11. $\infty$

12. 4

13. $\ln 5 - \ln 3$

14. $\frac{1}{2}$

15. 0

16. 0

17. 1

18. $e^{ab}$

## A.8 Section 11.2

1. $\dfrac{15}{8}\left(x^2+1\right)^{4/3} + C$

2. $\frac{2}{3}\sin\left(\sqrt{3x+2}\right)+C$

3. $\tan(\ln x)+C$

4. $\frac{3}{2}\ln(1+x^2)+7\arctan x+C$

5. $\frac{1}{2}\arctan(e^{2x})+C$

6. $-\frac{1}{5}\csc(5x^4)+C$

7. $\frac{1}{4}\ln\left|x^4+8x\right|+C$

8. $\frac{1}{2}\sin(x^2+1)+\arctan x+C$

9. $\frac{1}{8}\ln\left|8x+9\right|+C$

10. $\frac{1}{4}\arctan(2x)+C$

11. $\frac{1}{2}(\arcsin x)^2+C$

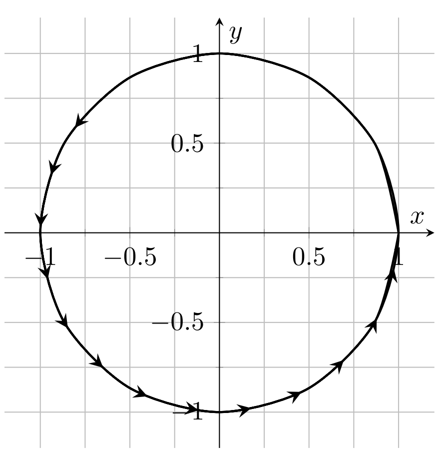
12. $e^{e^x}+C$

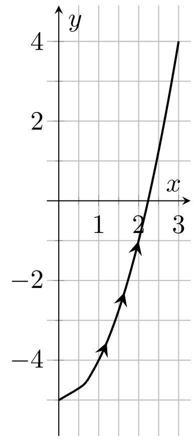
13. $-\cos^4 x-2\cos^3 x+8\cos x+C$

14. $1-\cos 1$

15. $\frac{3^5}{4}$

## A.9 Section 12.2

1. 


2. 


3. $x=2+4\cos t, y=3+4\sin t; 0\le t<2\pi$

4. $x=10\cos t, y=6\sin t, 0\le t<2\pi$

5. $x=t-1, y=4t-3$

6. $x=t, y=t^2$

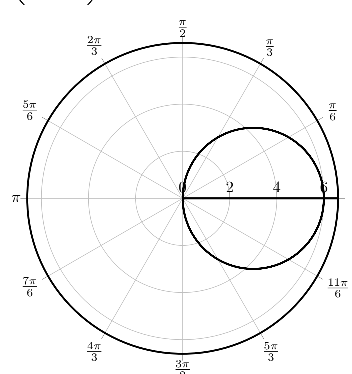
## A.10 Section 13.2

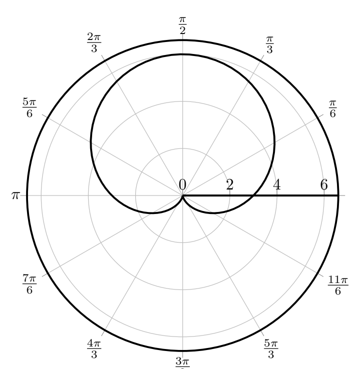
1. $(3,\sqrt{3})$

2. $(-4\sqrt{3},-4)$

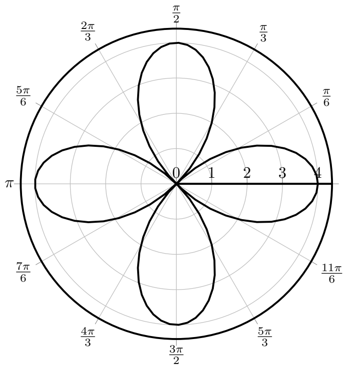
3. $\left(5\sqrt{2},\frac{3\pi}{4}\right)$

4. $\left(2,\frac{7\pi}{6}\right)$

5. 

6. 

7.



$\frac{\pi}{2}$
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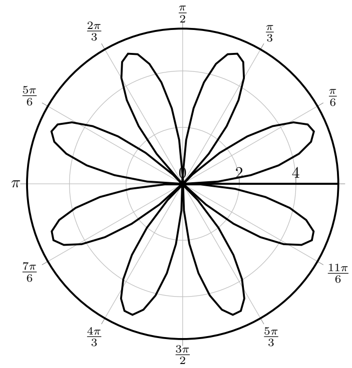
8.



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$\frac{5\pi}{3}$
$\frac{3\pi}{2}$

## B Recap of Basic Integration Rules

Let $a > 0$

1. $\int kf(u)du = k\int f(u)du$
2. $\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$
3. $\int du = u + C$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C; n \neq -1$
5. $\int \frac{1}{u} du = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int a^u du = \frac{1}{\ln a} a^u + C$
8. $\int \sin u du = -\cos u + C$
9. $\int \cos u du = \sin u + C$
10. $\int \sec u du = \ln|\sec u + \tan u| + C$
11. $\int \csc u du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u du = \tan u + C$
13. $\int \csc^2 u du = -\cot u + C$
14. $\int \sec u \tan u du = \sec u + C$
15. $\int \csc u \cot u du = -\csc u + C$
16. $\int \tan u du = -\ln|\cos u| + C$
17. $\int \cot u du = \ln|\sin u| + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$