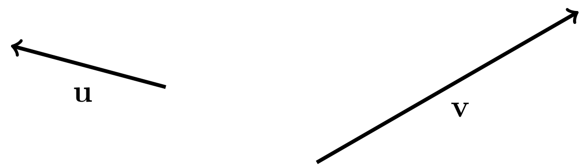
MAT 397: Exam 1 **Name:** ______________________________
Spring – 2020
02/12/2020
80 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 10 pages (including this cover page) and 9 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	15	
3	17	
4	15	
5	12	
6	15	
7	16	
8	10	
9	10	
Total:	120	

1. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors shown below.



Sketch the following vectors.

(a) (2 points) $-\frac{1}{2}\mathbf{v}$

(b) (2 points) $\mathbf{u}+\mathbf{v}$

(c) (3 points) $\mathbf{v}-\mathbf{u}$

(d) (3 points) $\mathrm{proj}_{\mathbf{v}}\mathbf{u}$

2. Let $\mathbf{u} = \langle 2, -2, 1\rangle$ and $\mathbf{v} = \langle 1, 1, 3\rangle$.

   (a) (3 points) Compute $2\mathbf{v} - \mathbf{u}$.

   (b) (3 points) Are $\mathbf{u}$ and $\mathbf{v}$ parallel? Explain.

   (c) (5 points) Find a vector perpendicular to *both* $\mathbf{u}$ and $\mathbf{v}$.

   (d) (4 points) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

3. Let $\mathbf{u} = \langle 2, -2, 1 \rangle$ and $\mathbf{v} = \langle 1, 1, 3 \rangle$.

   (a) (4 points) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

   (b) (3 points) Find a unit vector which points in the same direction as $\mathbf{u}$.

   (c) (3 points) Compute $\mathbf{u} \cdot \mathbf{v}$.

   (d) (2 points) Are $\mathbf{u}$ and $\mathbf{v}$ perpendicular? Explain.

   (e) (5 points) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

4. Complete the following parts:

   (a) (5 points) Find the line which passes through the point $(1, 3, -1)$ in the direction of $-\mathbf{i} + 2\mathbf{k}$.

   (b) (5 points) Find the equation of the plane with normal vector $\langle 3, -1, 5\rangle$ and which contains the point $(-2, 9, 3)$.

   (c) (5 points) Find the line perpendicular to the plane $3x - 2y + z = 6$ and which passes through the point where the plane intersects the x-axis.

5. Consider the lines $r_1(t) = (1 - t, t, t + 1)$ and $r_2(t) = (-1 - 2t, 2t, 2t + 5)$.

   (a) (6 points) Show that $r_1(t)$ and $r_2(t)$ are parallel.

   (b) (6 points) Find the distance between $r_1(t)$ and $r_2(t)$.

6. Suppose a subatomic particle has velocity vector $\mathbf{v}(t) = \left\langle \frac{1}{t+1},\, 2t,\, e^{-t} \right\rangle$.

(a) (4 points) Find $\mathbf{a}(t)$.

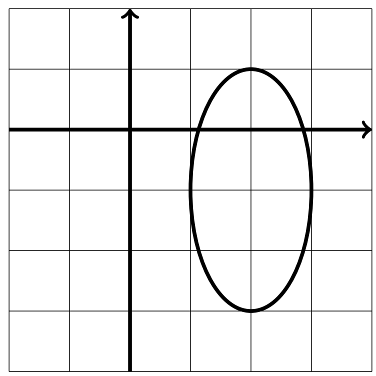
(b) (6 points) If $\mathbf{r}(0) = \langle 0, 1, 2 \rangle$, find $\mathbf{r}(t)$.

(c) (5 points) Set up completely, *but do not evaluate*, an integral which computes how far the particle travels between $t = 0$ and $t = 1$.

7. Find parametrizations for the following curves (specify the domain if necessary):

   (a) (4 points) The line segment pointing from $(1, -1, 2)$ to $(3, 2, 1)$.

   (b) (4 points) The curve shown below, oriented counterclockwise.



   (c) (4 points) The curve given by $x = \cos y + e^{\sin y}$.

   (d) (4 points) The curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x - 2y + z = 1$.

8. (10 points) Identify the following curves/surfaces in $\mathbb{R}^3$:

(a) ________________________ : $x^2 + y^2 + z^2 = 6$

(b) ________________________ : $x^2 - 3 = 2y^2 + \frac{1}{3}z^2$

(c) ________________________ : $y^2 = 2x^2 + 3z^2$

(d) ________________________ : $y = x + z$

(e) ________________________ : $y^2 - 4 = z^2 - 9x^2$

(f) ________________________ : $y = x^2 - 4$

(g) ________________________ : $2x^2 + 3y^2 + 4z^2 = 5$

(h) ________________________ : $z = \frac{1}{2}x^2 + \frac{1}{3}y^2$

(i) ________________________ : $y + 2z^2 - x^2 = 0$

(j) ________________________ : $(2t + 1, 1 - t, 4t)$

9. (10 points) Use the method of level curves, with slices/cross-sections in at least two directions, to sketch the surface $4x^2 + z^2 - y^2 = 1$.