MAT 397: Exam 2 **Name:** ______________________________
Spring – 2020
04/03/2020
180 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 14 pages (including this cover page) and 11 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	15	
2	16	
3	15	
4	15	
5	15	
6	16	
7	16	
8	16	
9	16	
10	15	
11	15	
Total:	170	

1. Determine the following limits. If the limit exists, compute its value. If the limit does not exist, prove it.

   (a) (3 points) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{x-2y}{2x^2+3y}$

   (b) (4 points) $\displaystyle\lim_{(x,y,z)\to(2,-1,2)} \frac{x-z}{x^2+xy-xz-yz}$

(c) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2+2y^2}$

(d) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^3 \sin x}{x^4+y^2}$

2. Let $f(x, y, z) := \dfrac{xe^{xy} + y^x - z^2}{1 - z}$. Find the following:

(a) (4 points) $f_x =$

(b) (4 points) $f_y =$

(c) (4 points) $f_z =$

(d) (4 points) $f_{yz} =$

3. (15 points) Define the following functions:

$$f(x,y) = \frac{x^2}{y}$$

$$x(m) = \tan(\ln m) + 3 \qquad m(s,t) = 3 - t\sqrt{s}$$

$$y(n) = n^2 - \arctan n - 1 \qquad n(s,t) = 2 - te^{\sin(t-2s)}$$

Compute $\dfrac{\partial f}{\partial t}$ when $(s,t) = (1,2)$.

4. Let S be the surface defined by $z = x - y^2$.

(a) (10 points) Find the equation of the tangent plane to S at $(2, -1, 1)$.

(b) (1 point) Find a direction perpendicular to S at $(2, -1, 1)$.

(c) (4 points) Find the equation of the normal line to S at $(2, -1, 1)$.

5. Let $f(x, y, z) := x^{10} + 2\sqrt{y} - e^{4-z}$.

(a) (3 points) Compute $f(1, 9, 4)$.

(b) (6 points) Find the total differential for $f(x, y, z)$.

(c) (6 points) Approximate $(0.99)^{10} + 2\sqrt{8.7} - e^{4.1}$.

6. The temperature in the region of a research facility is given by the function

$$f(x,y) = \frac{x^2 + y + e^{x+2y}}{3}$$

where x is the number of miles East/West ($+\hat{x}/-\hat{x}$) from the facility and y is number of miles North/South ($+\hat{y}/-\hat{y}$) from the facility. Suppose you are at a point 4 miles East and 2 mile South of the facility, i.e. $(x,y) = (4,-2)$.

(a) (6 points) What is the rate of change in the temperature if you are hiking straight towards the facility?

(b) (2 points) At your current position, what direction does the temperature increase most rapidly?

(c) (2 points) At your current position, what direction does the temperature decrease most rapidly?

(d) (3 points) Approximately how far must you travel in the direction you gave in (c) to see a decrease of $\frac{1}{\sqrt{5}}$ in the temperature?

(e) (3 points) At your current position, what is a direction you can travel so that the temperature does not change?

7. (16 points) Find and classify all critical points for the function $f(x, y) = e^{-x}(x^2 + 3y^2)$.

8. (16 points) An exoplanet's orbit about its star is given by $x^2+y^2=1$, i.e. the exoplanet has a circular orbit. The planet has 12 moons, and number of moons on the dark side of the planet at a point (x,y) in its orbit is given by $f(x,y)=x^2+4xy+y^2+4$. Find the greatest and fewest number of moons you could see at night on this exoplanet.

9. Complete the following parts:

   (a) (8 points) $\displaystyle\int_{-2}^{0}\int_{x^2}^{4} x\sin(y^2)\;dy\;dx$

   (b) (8 points) Compute the volume bounded by $x + 2y + 3z = 4$ and the coordinate planes by using a triple integral.

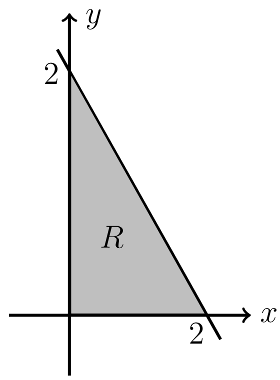
10. Complete the following parts—you need not evaluate any of the integrals:

   (a) (5 points) Change the following integral to polar coordinates: $\int_0^1 \int_y^1 \frac{y}{\sqrt{x^2+y^2}}\, dx\, dy$

   (b) (5 points) Change the following integral to cylindrical coordinates:
   $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\, dz\, dx\, dy$

   (c) (5 points) Change the following integral to spherical coordinates: $\iiint_R z\ dV$, where R is the bounded by $x = \sqrt{y^2+z^2}$ and the sphere $x^2+y^2+z^2=9$

11. (15 points) A steel plate has density function $\delta(x, y) = x + y$, and shape given by the region R given below:



Find the total mass and the center of mass for this steel plate.