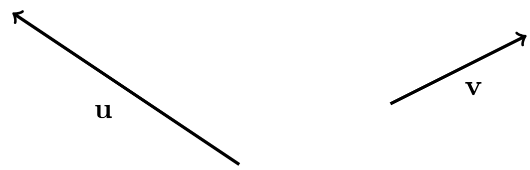
## Vector Geometry & Algebra

**Problem 1:** Given the vectors




Sketch the following vectors:

(a) $2\mathbf{v}$

(b) $-\frac{1}{3}\mathbf{u}$

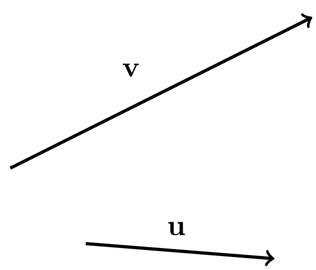
(c) $\mathbf{v}-\mathbf{u}$

(d) $\mathbf{u}+2\mathbf{v}$

(e) $\textbf{proj}_{\mathbf{v}}\,\mathbf{u}$

(f) $\mathbf{u}\times\mathbf{v}$

**Problem 2:** Given the vectors




Sketch the following vectors:

(a) $2\mathbf{v}$

(b) $-\frac{1}{3}\mathbf{u}$

(c) $\mathbf{v}-\mathbf{u}$

(d) $\mathbf{u}+2\mathbf{v}$

(e) $\textbf{proj}_{\mathbf{v}}\,\mathbf{u}$

(f) $\mathbf{u}\times\mathbf{v}$

**Problem 3:** For the vectors in Problem 1, which is larger: $\mathbf{u}\cdot\mathbf{u}$ or $\mathbf{v}\cdot\mathbf{v}$? Explain.

**Problem 4:** Determine if the vector $\langle 2,-4,10\rangle$ is parallel to $\langle -1,2,-5\rangle$.

**Problem 5:** Determine if the vector $\langle 1,0,-3\rangle$ is parallel to the vector $\langle -2,0,6\rangle$.

**Problem 6:** Determine if the vector $\langle 1,2,0\rangle$ is parallel to the vector $\langle 0,1,2\rangle$.

**Problem 7:** Let $\mathbf{u}=\langle 1,-1\rangle$, $\mathbf{v}=\langle -2,3\rangle$, and $\mathbf{w}=\langle 4,0\rangle$, find the following and if the result is a vector, sketch the vector:

(a) $2\mathbf{u}$

(b) $-\mathbf{w}$

(c) $2\mathbf{u}-3\mathbf{v}$

(d) $\|\mathbf{u}+\mathbf{w}\|$

(e) $\mathbf{u} - \mathbf{v} + \mathbf{w}$

(f) $\|\mathbf{w}\|\mathbf{u}$

**Problem 8:** Let $\mathbf{u} = \langle 1, 3, -2\rangle$, $\mathbf{v} = \langle 1, 1, 1\rangle$, and $\mathbf{w} = \langle 0, -1, 5\rangle$, find the following and if the result is a vector, sketch the vector:

(a) $2\mathbf{u}$

(b) $-\mathbf{w}$

(c) $2\mathbf{u} - 3\mathbf{v}$

(d) $\|\mathbf{u} + \mathbf{w}\|$

(e) $\mathbf{u} - \mathbf{v} + \mathbf{w}$

(f) $\|\mathbf{w}\|\mathbf{u}$

**Problem 9:** Given $\mathbf{u} = 2\,\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}$, find a unit vector which points in the opposite direction of $\mathbf{u}$. Find also a unit vector which points in the same direction of $\mathbf{u}$.

**Problem 10:** Find a vector parallel to $\mathbf{i} + \mathbf{j} - \mathbf{k}$ with length 5.

**Problem 11:** Find a vector which points from $(1, 2, 3)$ to $(1, 0, -1)$.

**Problem 12:** Find the displacement vector from the point $(1, 3)$ to the point $(-1, 5)$.

**Problem 13:** Determine if the points $(1, 1, 1)$, $(1, -1, 2)$, and $(3, 0, 2)$ are collinear.

**Problem 14:** Given a vector $\mathbf{w} = \langle -1, 7\rangle$, find scalars $a, b$ so that $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, where $\mathbf{u} = \langle 1, 2\rangle$ and $\mathbf{v} = \langle 1, -1\rangle$.

**Problem 15:** If $\|\mathbf{u}\| = 6$, what is $\|c\mathbf{u}\|$, where $c = -2$?

**Problem 16:** If $\|\mathbf{u}\| = 2$ and $\mathbf{u}$ makes an angle of $60°$ with the positive $x$-axis, write $\mathbf{u}$ in the form $\mathbf{u} = a\,\mathbf{i} + b\,\mathbf{j}$.

**Problem 17:** If $\|\mathbf{u}\| = 3$ and $\mathbf{u}$ makes an angle of $30°$ with the positive $y$-axis, write $\mathbf{u}$ in the form $\mathbf{u} = a\,\mathbf{i} + b\,\mathbf{j}$.

**Problem 18:** Let $f(x) = 5 - x^2$. Find a vector which is parallel to the graph of $f(x)$ at $(1, 4)$. Find also a vector which is perpendicular to the graph of $f(x)$ at $(1, 4)$.

**Problem 19:** Suppose that $\mathbf{u} = a\,\mathbf{i} + b\,\mathbf{j}$ is a unit vector. Is it then true that the point $(a, b)$ lies on the unit circle?

**Problem 20:** Suppose that $\mathbf{u} = a\,\mathbf{i} + b\,\mathbf{j} = \mathbf{0}$. Is it then true that $a = -b$?

**Problem 21:** Suppose that $a = b$, is it true that $\|a\,\mathbf{i} + b\,\mathbf{j}\| = \sqrt{2}a$.

**Problem 22:** Suppose that $\mathbf{u} = \mathbf{i} + \sqrt{3}\,\mathbf{j}$. Find the angle $\mathbf{u}$ makes with the positive $y$-axis.

**Problem 23:** Suppose a 50 lb weight is suspended by two cables which make angles of $50°$ and $30°$ with the ceiling, respectively. Find the tension vectors for each cable, as well as the total tension in each cable.

**Problem 24:** If $\mathbf{u}$ and $\mathbf{v}$ are vectors, prove that $|\mathbf{u}+\mathbf{v}| \leq |\mathbf{u}| + |\mathbf{v}|$. Prove this algebraically and geometrically.

## Dot & Cross Product

**Problem 25:** Let $\mathbf{u} = \langle 1, -3 \rangle$ and $\mathbf{v} = \langle -2, 1 \rangle$. Compute the following:

(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $\mathbf{u} \cdot \mathbf{u}$

(c) $2\mathbf{u} \cdot \mathbf{v}$

(d) $\mathbf{u} \cdot 3\mathbf{v}$

(e) $2\mathbf{u} \cdot 3\mathbf{v}$

**Problem 26:** Let $\mathbf{u} = \langle 1, -1, 1 \rangle$ and $\mathbf{v} = \langle 2, 0, 1 \rangle$. Compute the following:

(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $\mathbf{u} \cdot \mathbf{u}$

(c) $2\mathbf{u} \cdot \mathbf{v}$

(d) $\mathbf{u} \cdot 3\mathbf{v}$

(e) $2\mathbf{u} \cdot 3\mathbf{v}$

**Problem 27:** Let $\mathbf{u} = \langle 1, -1, 1 \rangle$, $\mathbf{v} = \langle 2, 3, 1 \rangle$, and $\mathbf{w} = \langle 2, 0, 1 \rangle$. Is $\mathbf{u} \perp \mathbf{v}$? Is $\mathbf{u} \perp bw$? Is $\mathbf{v} \perp \mathbf{w}$? Explain.

**Problem 28:** Find at least four vectors perpendicular to the vector $\mathbf{u} = 2\,\mathbf{i} - 3\,\mathbf{j} + 2\,\mathbf{k}$.

**Problem 29:** Find at least 3 pairs $(a, b)$ so that the vector $\mathbf{u} = \langle a, -2, b \rangle$ is perpendicular to the vector $\mathbf{v} = \langle 3, -1, 4 \rangle$.

**Problem 30:** Find $\mathbf{u} \cdot \mathbf{v}$ if $\|\mathbf{u}\| = 8$, $\|\mathbf{v}\| = 5$, and the angle between the vectors is $45°$.

**Problem 31:** Find the angle between the vectors $\mathbf{u} = \mathbf{i} + \mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 2\,\mathbf{j}$.

**Problem 32:** Find the angle between the vectors $\mathbf{u} = \mathbf{i} - \mathbf{k}$ and $\mathbf{v} = 2\,\mathbf{i} - \mathbf{j}$.

**Problem 33:** Use the dot product to compute the length of $\mathbf{u} = \langle -1, 3, 4 \rangle$.

**Problem 34:** Given $\mathbf{u} = \langle 1, -3 \rangle$, find the angle $\mathbf{u}$ makes with the positive $x$ and $y$ axes.

**Problem 35:** Given $\mathbf{u} = \langle 1, -3, 1 \rangle$, find the angle $\mathbf{u}$ makes with the positive $x$, $y$, and $z$ axes.

**Problem 36:** Determine if the triangle with vertices $(2, -7, 3)$, $(-1, 5, 8)$, and $(4, 6, -1)$ is acute, obtuse, or right. Is the triangle an equilateral or isosceles triangle?

**Problem 37:** Find the acute angle between $(y+1)^2 = x$ and $y = x^3 - 1$ at their point(s) of intersection.

**Problem 38:** Find a pair of vectors that point in opposite directions that are both perpendicular to $\mathbf{u} = \langle 2, -3 \rangle$. Can these be forced to be unit vectors? Are there infinitely many such vectors? If there are, do they all have to be parallel?

**Problem 39:** Find the angles the diagonal of a cube makes with its edges and faces.

**Problem 40:** Find the acute angle between the lines $x + 2y = 7$ and $5x - y = 2$.

**Problem 41:** A force vector $\mathbf{F} = 2\,\mathbf{i} - 3\,\mathbf{j} + 2\,\mathbf{k}$ acts on a body, pushing it straight from $(1, 0, 1)$ to $(-3, 1, 2)$. Find the work done.

**Problem 42:** Find the values of $x$ such that the angle between $\langle 1, 1, 1 \rangle$ and $\langle x, 2, -1 \rangle$ is $60°$.

**Problem 43:** Let $\mathbf{u} = \langle 1, 3 \rangle$ and $\mathbf{v} = \langle 1, -1 \rangle$. Find $\mathbf{proj_u}\,\mathbf{v}$ and $\mathbf{proj_v}\,\mathbf{u}$. Use these vectors to determine if $\mathbf{u} \perp \mathbf{v}$.

**Problem 44:** Let $\mathbf{u} = \langle 1, -4, 1 \rangle$ and $\mathbf{v} = \langle 5, 0, -1 \rangle$. Find $\mathbf{proj_u}\,\mathbf{v}$ and $\mathbf{proj_v}\,\mathbf{u}$. Use these vectors to determine if $\mathbf{u} \perp \mathbf{v}$.

**Problem 45:** Compute the determinant

$$\begin{vmatrix} 1 & 3 \\ -5 & 2 \end{vmatrix}$$

**Problem 46:** Compute the determinant

$$\begin{vmatrix} 0 & \pi \\ 6 & \pi^3 \end{vmatrix}$$

**Problem 47:** Compute the determinant

$$\begin{vmatrix} 5 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$$

**Problem 48:** Compute the determinant

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 5 \end{vmatrix}$$

**Problem 49:** Let $\mathbf{u} = \langle 1, 3 \rangle$ and $\mathbf{v} = \langle -1, 2 \rangle$. Find a vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$. Can you find infinitely many such vectors? If not, explain why, and if it is possible, find at least 3 more such vectors.

**Problem 50:** Let $\mathbf{u} = \langle 1, 0, -2 \rangle$ and $\mathbf{v} = \langle 5, -1, 0 \rangle$. Find a vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$. Can you find infinitely many such vectors? If not, explain why, and if it is possible, find at least 3 more such vectors.

**Problem 51:** Find a unit vector which is perpendicular to both $\mathbf{u} = \langle 1, 0, -1 \rangle$ and $\mathbf{v} = \langle 0, 1, 2 \rangle$. Is this the only possible unit vector with this property?

**Problem 52:** Find the area of the parallelogram spanned by $\mathbf{u} = 2\,\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 5\,\mathbf{i} + 2\,\mathbf{j}$.

**Problem 53:** Find the area of the parallelogram spanned by $\mathbf{u} = \mathbf{i} + 3\,\mathbf{j}$ and $\mathbf{v} = -\,\mathbf{i} + \mathbf{j}$.

**Problem 54:** Find the area of the parallelogram spanned by $\mathbf{u} = \langle 1, 1, 1 \rangle$ and $\mathbf{v} = \langle 3, -1, 2 \rangle$.

**Problem 55:** Find the area of the triangle with vertices $(1, 1)$, $(2, 3)$, and $(-5, -1)$.

**Problem 56:** Find the area of the triangle with vertices $(1, 0, 3)$, $(-1, 2, 5)$, and $(0, 0, -1)$.

**Problem 57:** Show that the quadrilateral with vertices $(5, 2, 0)$, $(2, 6, 1)$, $(2, 4, 7)$, and $(5, 0, 6)$ is a parallelogram. Sketch this parallelogram and find its area.

**Problem 58:** Use the cross product to determine if the vectors $\langle 1, 1, 1 \rangle$ and $\langle 3, 3, 3 \rangle$ are parallel.

**Problem 59:** Find the angle between $\langle 1, 0 \rangle$ and $\langle 1, -3 \rangle$ using the cross product. Use the cross product to determine the angle between $\langle 1, 1, 1 \rangle$ and $\langle 1, -2, 1 \rangle$.

**Problem 60:** Find the volume of the parallelepiped spanned by $3\,\mathbf{i} - 5\,\mathbf{j} + \mathbf{k}$, $2\,\mathbf{j} - 2\,\mathbf{k}$, and $3\,\mathbf{i} + \mathbf{j} + \mathbf{k}$.

**Problem 61:** Are the vectors $\langle 1, 4, -7 \rangle$, $\langle 2, -1, 4 \rangle$, and $\langle 0, -9, 18 \rangle$ coplanar? Explain.

**Problem 62:** Find $|\mathbf{u} \times \mathbf{v}|$ if $\|\mathbf{u}\| = 2$, $\|\mathbf{v} = 4\|$, and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $30°$.

**Problem 63:** If $\mathbf{u}$ and $\mathbf{v}$ are vectors, what is $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v})$? What is $2\mathbf{u} \times \mathbf{u}$? Explain.

**Problem 64:** Determine if the vectors $\mathbf{i} + 5\,\mathbf{j} - 2\,\mathbf{k}$, $3\,\mathbf{i} - \mathbf{j}$, and $5\,\mathbf{i} + 9\,\mathbf{j} - 4\,\mathbf{k}$ are coplanar.

**Problem 65:** Is the following statement true or false: if $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

**Problem 66:** Is it always the case that $\mathbf{u} \times \mathbf{u} = \mathbf{0}$? Explain.

**Problem 67:** If $\mathbf{u} \cdot \mathbf{v} = \sqrt{3}$ and $\mathbf{u} \times \mathbf{v} = \langle 1, 2, 2 \rangle$, what is teh angle between $\mathbf{u}$ and $\mathbf{v}$?

**Problem 68:** Find all vectors $\mathbf{u}$ so that $\langle 1, 2, 1 \rangle \times \mathbf{u} = \langle 3, 1, -5 \rangle$. Is there a vector $\mathbf{u}$ so that $\langle 1, 2, 1 \rangle \times \mathbf{u} = \langle 3, 1, 5 \rangle$.

**Problem 69:** Prove that if $\mathbf{u}$ and $\mathbf{v}$ are vectors that $|\mathbf{u}+\mathbf{v}|^2 + |\mathbf{u}-\mathbf{v}|^2 = 2(|\mathbf{u}|^2 + |\mathbf{v}|^2)$.

**Problem 70:** Prove the Law of Cosines: if a triangle has sides $a, b, c$, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

where $\theta$ is the angle opposite $c$.

**Problem 71:** Prove that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a square.

**Problem 72:** Prove that if $P_1, P_2, \ldots, P_n$ are vertices of a regular polygon with $n$-sides and $\mathcal{O}$ is the center of the polygon, that $\sum_{i=1}^{n} \vec{\mathcal{O}P_i} = \mathbf{0}$.

**Problem 73:** Find the vector, parametric, and symmetric forms of the line $\ell(t) = \langle 1, -1, 2\rangle t + (1, 2, 3)$.

**Problem 74:** Find the vector, parametric, and symmetric forms of the line

$$\begin{cases} x = 2t + 1 \\ y = 5 \\ z = 3 - 2t \end{cases}$$

**Problem 75:** Find the vector, parametric, and symmetric forms of the line

$$\frac{x-1}{3} = \frac{2y-1}{4} = \frac{z+1}{5}$$

**Problem 76:** Find the equation of the line parallel to the vector $\langle 1, 3, 1\rangle$ through the point $(1, -1, 2)$.

**Problem 77:** Find the equation of the line through $(1, 3, -2)$ and $(5, 2, 4)$.

**Problem 78:** Find the (parametric) equation of the line through $(1, 3)$ and $(-5, 4)$.

**Problem 79:** Find the equation of the line perpendicular to the plane $2x - y + 3z = 5$ that passes through the point $(5, 0, -2)$.

**Problem 80:** Find the equation of the line passing through the origin that is perpendicular to both $\mathbf{i} + 2\,\mathbf{j}$ and $\mathbf{i} - 3\,\mathbf{k}$.

**Problem 81:** Show that the lines $r(t) = (4t+3, 6t+3, 2t)$ and $s(t) = (2t, 3t-2, t-1)$ are the same.

**Problem 82:** Show that the lines $r(t) = \langle 2, -1, 3\rangle t + (0, 1, 1)$ and $s(t) = \langle 4, -2, 6\rangle t + (1, 1, -1)$ are parallel.

**Problem 83:** Show that the lines $r(t) = (t+4, t+3, 1-t)$ and $s(t) = (6t+3, 2-4t, 2t+2)$ are perpendicular.

**Problem 84:** Show that the lines $r(t) = (3t+1, t, 2t+1)$ and $s(t) = (3t-4, 6t-10, 3t-4)$ intersect.

**Problem 85:** Show that the lines $r(t) = (t+1, 2-t, 2t+1)$ and $s(t) = (t, 2t, 1-t)$ are skew.

**Problem 86:** Determine if the lines $r(t) = (2t, t-1, t)$ and $s(t) = (-2-t, 3t+5, 2t+4)$ intersect, are skew, perpendicular, parallel, or the same.

**Problem 87:** Determine if the lines $r(t) = (t+1, 2-t, 3t)$ and $s(t) = (s+5, 4-s, 3s+2)$ intersect, are skew, perpendicular, parallel, or the same.

**Problem 88:** Determine if the lines $r(t) = (2t+1, 3-t, t+5)$ and $s(t) = (2t, t+1, 1-t)$ intersect, are skew, perpendicular, parallel, or the same.

**Problem 89:** Determine if the lines $r(t) = (2t-1, t+1, 3t+1)$ and $s(t) = (5t-1, 1-t, 1-3t)$ intersect, are skew, perpendicular, parallel, or the same.

**Problem 90:** Determine if the lines $r(t) = (2t+1, 1, -1-t)$ and $s(t) = (4t+5, 1, -3-2t)$ intersect, are skew, perpendicular, parallel, or the same.

**Problem 91:** Determine the equation of the plane passing through $(1, 2, 3)$ with normal vector $\mathbf{n} = \langle 2, -1, 3 \rangle$.

**Problem 92:** Determine the equation of the plane that is perpendicular to the vectors $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and passes through the point $(1, 0, 1)$.

**Problem 93:** Determine the equation of the plane passing through the points $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 0)$.

**Problem 94:** Determine the equation of the plane which is perpendicular to the planes $x+y+z = 3$ and $2x + y - z = -2$ and contains the point $(1, 1, -1)$.

**Problem 95:** Find an equation of the plane passing through $(0, 1, 2)$ that is parallel to the $xz$-plane. Is this plane unique? Explain.

**Problem 96:** Find an equation of a plane containing the $x$-axis and making an angle of $\frac{\pi}{3}$ with the positive $y$-axis.

**Problem 97:** Find the equation of the plane with $x, y, z$ intercepts $a, b, c$, respectively.

**Problem 98:** Determine the equation of the plane containing $(2, 2, 2)$ and perpendicular to the line

$$\begin{cases} x = 2t + 1 \\ y = 1 - 5t \\ z = t + 1 \end{cases}$$

**Problem 99:** Determine the equation of the plane containing the line $\ell(t) = (t+1, 1-t, t+2)$ and parallel to the plane $3x + 4y - z = 2$.

**Problem 100:** Determine the equation of the plane passing through the line of intersection between $x - z = 1$ and $y + 2z = 3$ and is perpendicular to $x + y - 2z = 1$.

**Problem 101:** Determine where (if anywhere) the line $\ell(t) = (t + 1, 3t + 1, t)$ intersects the plane $x + y + z = 3$.

**Problem 102:** Determine where (if anywhere) the line $\ell(t) = (t + 1, 4 - t, t)$ intersects the plane $2x + 3y + z = 0$.

**Problem 103:** Determine where (if anywhere) the line $\ell(t) = (4 - t, 3 + t, 2t)$ intersects the plane $x - y + 3z = 5$.

**Problem 104:** Find parametric equations for the line of intersection between the planes $x+y-z = 4$ and $2x - y + 3z = 4$.

**Problem 105:** Determine if the plane $x + 2y - 3z = 4$ contains the line $\ell(t) = (t + 6, 1 - t, t)$.

**Problem 106:** Determine if the plane $x - y + z = 2$ contains the line $\ell(t) = (2t - 1, 3t + 1, t + 4)$.

**Problem 107:** Are the planes $x+y+z = 5$ and $x-y+z = 6$ parallel, intersecting, or perpendicular?

**Problem 108:** Are the planes $2x - z = 4$ and $x + 2z = 6$ parallel, intersecting, or perpendicular?

**Problem 109:** Are the planes $x - 2y + z = 5$ and $-2x + 4y - 2z = 2$ parallel, intersecting, or perpendicular?

**Problem 110:** Determine the angle of intersection between the planes $x - 2y + z = 2$ and $x + 2y - 4z = 1$.

**Problem 111:** Find the angle of intersection between the planes $x-2y+z = 0$ and $2x+3y-2z = 0$.

**Problem 112:** Determine a method to determine if a given point $P$ lies on, 'above', or 'below' a given plane.

## Distance Problems

**Problem 113:** Find the projection of the point $P = (1, -3, 2)$ to the $xy$, $yz$, and $xz$-planes. Use this to determine the distance from $P$ to these planes.

**Problem 114:** Sketch the point $P = (-1, -3, -2)$ in $\mathbb{R}^3$. Then find the distance from $P$ to…

(a) $xy$-plane

(b) $yz$-plane

(c) $xz$-plane

(d) $x$-axis

(e) $y$-axis

(f) $z$-axis

(g) the origin

(h) the point $(1, 4, 2)$

**Problem 115:** Sketch the point $P = (1, -5, 2)$ in $\mathbb{R}^3$. Then find the distance from $P$ to...

(a) $xy$-plane

(b) $yz$-plane

(c) $xz$-plane

(d) $x$-axis

(e) $y$-axis

(f) $z$-axis

(g) the origin

(h) the point $(1, 4, 2)$

**Problem 116:** Find the distance from the point $(3, -1, 2)$ to the line $\ell(t) = (2t - 1, t - 2, t - 1)$.

**Problem 117:** Find the distance from the plane $x + y - z = 5$ to the plane $2x - y + 3z = 4$.

**Problem 118:** Find the distance between the lines $r(t) = \langle 2, -2, 2\rangle\ t + (-1, 0, 1)$ and $s(t) = \langle 1, 2, 1\rangle\ t + (-1, 1, -2)$.

**Problem 119:** Find the distance from the line $r(t) = (2t + 1, t + 1, 1 - 2t)$ to the line $x(t) = (2t, t - 1, 2 - 2t)$.

**Problem 120:** Find the distance between the lines $r(t) = (t+3, 3t+6, t+1)$ and $x(t) = (2t - 1, 1 - t + 2t - 3)$.

**Problem 121:** Find the distance from the line $x(t) = (6t + 4, -2t, 4t + 3)$ and $s(t) = (3t - 8, 4 - t, 2t - 5)$.

**Problem 122:** Find the distance between the planes $x + y = 3$ and $x - z = 5$.

**Problem 123:** Find the distance from the point $(1, 1, 3)$ and the line

$$\begin{cases} x = 4t + 1 \\ y = -t - 1 \\ z = t + 1 \end{cases}$$

**Problem 124:** Find the distance between the planes $x - y + 3z = 17$ and $x - y + 3z = 6$.

**Problem 125:** Find the distance from the line $r(t) = \langle 5, 1, 1\rangle\ t + (4, 3, 1)$ to the line $s(t) = \langle 3, 2, 1\rangle\ t + (5, 6, 2)$.

**Problem 126:** Find the distance between the given lines

$$\begin{cases} x = t - 2 \\ y = 1 - 3t \\ z = t + 3 \end{cases} \qquad \begin{cases} x = t + 1 \\ y = 1 - 2t \\ z = t \end{cases}$$

**Problem 127:** Find the distance between the following lines

$$\begin{cases} x = t - 1 \\ y = t - 2 \\ z = t \end{cases} \qquad\qquad \begin{cases} x = 2t - 1 \\ y = 2t + 1 \\ z = 2t \end{cases}$$

**Problem 128:** Find the distance from the plane $x - 5y + z = -5$ and $x - 5y + z = -8$.

**Problem 129:** Find the distance from the line $r(t) = (2t+3, 1-t, t+1)$ to the sphere $x^2+y^2+z^2 = 1$. [Hint: Sketch the scenario and think about how you can reduce this to a case you already know how to do, making the necessary adjustments.]

**Problem 130:** Prove that the distance from a point $P \in \mathbb{R}^3$ to a line with direction vector $\mathbf{L}$ is given by

$$d = \frac{|\vec{PQ} \times \mathbf{L}|}{|\mathbf{L}|}$$

where $Q$ is any point on the line.

**Problem 131:** Prove that the distance between a point $P$ and a plane with normal vector $\mathbf{n}$ is given by

$$d = \frac{|\vec{PQ} \cdot \mathbf{n}|}{|\mathbf{n}|}$$

where $Q$ is any point on the plane.

**Problem 132:** Given lines $r(t) = P + t\mathbf{L}_1$ and $s(t) = Q + t\mathbf{L}_2$, show that the distance between them is

$$d = \frac{|\vec{PQ} \cdot (\mathbf{L}_1 \times \mathbf{L}_2)|}{|\mathbf{L}_1 \times \mathbf{L}_2|}$$

## Curves & Quadratic Surfaces

**Problem 133:** Identify the following curves/surfaces in $\mathbb{R}^3$, being as specific as possible:

(a) ________________________________: $x + 2y - z = 3$

(b) ________________________________: $z^2 - 4 = 2x^2 + 3y^2$

(c) ________________________________: $\frac{1}{2}x^2 + 5y^2 = 1 - z^2$

(d) ________________________________: $2t\,\mathbf{i} - \mathbf{j} + (1-t)\,\mathbf{k}$

(e) ________________________________: $y = x^2 + 2x + 1$

(f) ________________________________: $x + y^2 - z^2 = 0$

(g) ________________________________: $(t - 5, 1 + \cos t, 3 + \sin t)$

(h) ________________________________: $y = x^2 + z^2$

(i) ________________________________: $2x^2 - 3y^2 - z^2 = 0$

(j) ________________________________: $(2\cos t + 1, 4, 5\sin t - 2)$

(k) ________________________________: $z^2 + 4 = x^2 - y^2$

(l) ________________________________: $(1-t)(5,4,-1) + t(1,0,-1)$

**Problem 134:** Identify the following curves/surfaces in $\mathbb{R}^3$, being as specific as possible:

(a) ________________________________: $(2t+3, \cos 2t, \sin 2t + 4)$

(b) ________________________________: $y = 2x + 3$

(c) ________________________________: $\frac{5}{3}x^2 = \pi - y^2 + \frac{1}{2}z^2$

(d) ________________________________: $(y-1)^2 = x^2 + (z+3)^2$

(e) ________________________________: $(y-1)^2 - (z+1)^2 = 5 - x^2$

(f) ________________________________: $(\cos 8t, 3, \sin 8t + 5)$

(g) ________________________________: $(z-1)^2 = y$

(h) ________________________________: $\frac{1}{3}x = y^2 - 5(z+1)^2$

(i) ________________________________: $(x+1)^2 = 4 + y^2 + (z-3)^2$

(j) ________________________________: $(1-2t, 3, 4t+5)$

(k) ________________________________: $(1-t)(0,0,0) + t(1,2,3)$

(l) ________________________________: $1 - x = (z+2)^2 + (y-1)^2$

**Problem 135:** Identify the following curves/surfaces in $\mathbb{R}^3$, being as specific as possible:

(a) ________________________________: $(x-1) + (y+3) + (z-4) = 7$

(b) ________________________________: $(y-1)^2 = (z+1)^2 + (x-3)^2$

(c) ________________________________: $(y-1)^2 = x$

(d) ________________________________: $5 - y = 5x^2 + (z+1)^2$

(e) ________________________________: $(2\cos t, 3\sin t, 0)$

(f) ________________________________: $(2\cos t, 1-t, 3\sin t)$

(g) ________________________________: $\dfrac{(x-1)^2}{5} + 2y^2 + \dfrac{(z+3)^2}{4} = 1$

(h) ________________________________: $y^2 - x = (z-1)^2$

(i) ________________________________: $(1-t)(1,1,1) + t(\pi, 2\pi, 3\pi)$

(j) ________________________________: $(x-1)^2 - (y+1)^2 + (z+3)^2 = 6$

(k) ______________________________: $(y-1)^2 + (z-5)^2 = 3 + x^2$

(l) ______________________________: $(0, 0, t)$

**Problem 136:** Use level curves to sketch the surface $4x^2 + \frac{1}{9}y^2 + z^2 = 1$.

**Problem 137:** Use level curves to sketch the surface $4x^2 + y^2 - z^2 = 1$.

**Problem 138:** Use level curves to sketch the surface $y = (x-1)^2 + \frac{(z+1)^2}{2}$.

**Problem 139:** Use level curves to sketch the surface $y^2 - 2x^2 - z^2 = 1$.

**Problem 140:** Use level curves to sketch the surface $yz = 1$.

## Vector Functions

**Problem 141:** Find the domain of the vector function $r(t) = \sin t\,\mathbf{i} + t^2\,\mathbf{j} - e^t\,\mathbf{k}$.

**Problem 142:** Find the domain of the vector function $x(t) = \left\langle \sin e^t, \frac{1}{\sqrt{t}}, \sqrt{1-t} \right\rangle$.

**Problem 143:** Find the domain of the vector function $x(t) = \frac{1}{t^2-1}\,\mathbf{i} + \ln t\,\mathbf{j} + t^{-1/2}\,\mathbf{k}$.

**Problem 144:** Evaluate the vector valued function $r(t) = \sin(\pi t)\,\mathbf{i} - (2t^2+1)\,\mathbf{j} + \frac{t+1}{t-3}\,\mathbf{k}$ at times $t = -1, 0, 1, 2$.

**Problem 145:** Let $r(t) = \langle t, 2, t-1 \rangle$. Evaluate $r(t + \Delta t)$.

**Problem 146:** Evaluate the limit $\lim_{t \to 0} x(t)$, where $x(t) = \frac{\sin t}{t}\,\mathbf{i} + \sqrt{1-t}\,\mathbf{j} + \frac{1-e^t}{t}\,\mathbf{k}$.

**Problem 147:** Evaluate the limit $\lim_{t \to 1} r(t)$, where $r(t) = \left\langle \sqrt{t}, \frac{\ln t}{t^2-1}, \frac{1}{t-1} \right\rangle$.

**Problem 148:** Sketch the curves given by the following vector-valued functions:

(a) $r(t) = (\cos t + 1, \sin t - 3)$

(b) $r(t) = (3\cos t + 4, 2\sin t + 1)$

(c) $r(t) = (1-t)(1,2) + t(4,-1)$; $0 \le t \le 1$

(d) $r(t) = (1-t)(1,1,0) + t(-2,1,3)$; $0 \le t \le 1$

(e) $r(t) = (2\cos t + 1, 2\sin t, t)$

(f) $r(t) = (t, t^2, t^3)$

**Problem 149:** Parametrize the following plane curves:

(a) $y = 2x + 7$

(b) $x = 3$

(c) $y = \sin x$

(d) $x = \sin y$

(e) $(\ln y + 1)^3 = x$

(f) $(x - 1)^2 + y^2 = 9$

**Problem 150:** Determine the plane/space curve represented by the following vector-valued functions:

(a) $(2t + 1, t)$

(b) $(3, t)$

(c) $(t^2, t)$

(d) $(t^2, t + 1)$

(e) $(3\cos t, 3\sin t)$

(f) $(2\cos t, 3\sin t)$

(g) $(\cos t, \sin t, \sin 2t)$

**Problem 151:** Let $r(t) = \langle t, t^2 + 1, 1 - t \rangle$, and $x(t) = \langle t, 0, \sqrt{t} \rangle$. Find the following:

(a) $2r(t)$

(b) $r(t) + x(t)$

(c) $3r(t) - 2x(t)$

(d) $r(t) \cdot x(t)$

(e) $r(t) \times x(t)$

**Problem 152:** Prove that if $r(t)$ is a vector valued function and $r(t) \cdot r(t)$ is constant, that $r(t) \cdot r'(t) = 0$.

**Problem 153:** Consider the vector functions $r(t) = \langle 2, t + 1, t^2 \rangle$ and $x(t) = \langle 1, 2 - t, 1 \rangle$. Are $r(t)$ and $x(t)$ every perpendicular to each other? If so, where? Are they ever parallel to each other? If so, where?

**Problem 154:** If $r(t) = \sin 2t\, \mathbf{i} - t^{-1/2}\, \mathbf{j} + e^{\sqrt{t}}\, \mathbf{k}$, find $r'(t)$.

**Problem 155:** If $x(t) = \dfrac{2t+1}{1-3t}\,\mathbf{i} - e^t \sin t\,\mathbf{j} + (2+3t)^{10}\,\mathbf{k}$, find $x'(t)$.

**Problem 156:** If $r(t) = \left\langle \dfrac{\ln t}{t+1}, \sin e^{1-t}, \tan t \sec t \right\rangle$.

**Problem 157:** If $x(t) = \left\langle \dfrac{1}{t}, \dfrac{\sin\sqrt{t}}{\sqrt{t}}, \dfrac{1}{\sqrt[3]{t}} \right\rangle$, find $\displaystyle\int x(t)\,dt$.

**Problem 158:** If $r(t) = te^t\,\mathbf{i} - \dfrac{t-\sqrt{t}}{\sqrt{t}}\,\mathbf{j}$, find $\displaystyle\int r(t)\,dt$.

**Problem 159:** Given $x'(t) = \langle t, e^{2t}\rangle$ and $x(0) = \langle 0, 3\rangle$, find $x(t)$.

**Problem 160:** Given $r'(t) = 3t^2\,\mathbf{j} + 6\sqrt{t}\,\mathbf{k}$ and $r(0) = \mathbf{i} + 2\,\mathbf{j}$, find $r(t)$.

**Problem 161:** Given $r''(t) = \langle 0, -4\cos t, -3\sin t\rangle$, $r'(0) = 3\,\mathbf{k}$, and $r(0) = 4\,\mathbf{j}$, find $r(t)$.

**Problem 162:** Given $r''(t) = \mathbf{i} + \mathbf{k}$, $r'(1) = \mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$, and $r(0) = \mathbf{i} - \mathbf{k}$, find $r(t)$.

**Problem 163:** Given $a(t) = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $v(0) = \mathbf{0}$, and $r(0) = \mathbf{0}$, find $r(t)$.

**Problem 164:** Parametrize the circle with radius 3 centered at $(-1, 2)$, oriented counterclockwise. Sketch the curve.

**Problem 165:** Parametrize the line segment which goes from $(1, 2, 3)$ to $(-2, 1, 4)$. Sketch the curve.

**Problem 166:** Parametrize the ellipse centered at $(1, -3)$, with major axis parallel to the $y$-axis with length $6$ and semi-minor axis of length 2, oriented counterclockwise.

**Problem 167:** Parametrize the circle centered at $(1, 1)$ of radius 1, oriented clockwise.

**Problem 168:** Parametrize a helix centered around the point $(1, 2, 3)$, whose projection to the $yz$-plane is a circle of radius 3.

**Problem 169:** Parametrize a helix centered around $(1, 0, -1)$, whose projection to the $xz$-plane is an ellipse with semi-major and semi-minor axis lengths of 1 and 3, respectively.

**Problem 170:** Parametrize the curve of intersection between the given surfaces:

(a) $z = x^2 + y^2$, $x + y = 0$

(b) $z = x^2 + y^2$, $z = 4$

(c) $x^2 + y^2 = 4$, $z = x^2$

(d) $x^2 + 4y^2 + z^2 = 16$, $x = z^2$

(e) $x^2 + z^2 = 4$, $y^2 + z^2 = 4$ [In the first octant.]

**Problem 171:** Give parametrizations for the curve which traces out the triangle with vertices $(0, 0)$, $(2, 0)$, $(3, 1)$, oriented clockwise.

**Problem 172:** Give parametrizations for the curve which traces out the boundary of the curve formed by $x^2 + y^2 \leq 1$ with the Quadrant I.

**Problem 173:** Find the curve resulting from the intersection of the plane $x + y = 2$ with the paraboloid $z = 1 - x^2 + y^2$.

**Problem 174:** Find the curve resulting from the intersection of the plane $z - y = 3$ with the hyperbolic paraboloid $x = y^2 - 3z^2$.

**Problem 175:** Find equations for the surface resulting from revolving the curve $y = x^2$ about the $x$-axis.

**Problem 176:** Given $r(t) = \langle -t, 4t, 2t \rangle$, find the length of the curve given by $r(t)$ over the interval $[0, 3]$.

**Problem 177:** Given $x(t) = \mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, find the length of the portion of the curve from $x(0)$ and $x(1)$.

**Problem 178:** Find the length of the helix $r(t) = (2\cos t, 2\sin t, \frac{1}{2}t)$ located between the planes $z = 0$ and $z = 6$.

**Problem 179:** Find the curvature of the curve $r(t) = \langle t, 2\sin t, 2\cos t$.

**Problem 180:** Find the curvature of the curve $x(t) = t\,\mathbf{i} + t^2\,\mathbf{j}$.

**Problem 181:** Find the curvature of the curve $r(t) = \langle e^t, 4t \rangle$ at the point $(1, 0)$.

**Problem 182:** Find the $T, N, B$ frame for $r(t) = \langle t, 2\cos t, 2\sin t \rangle$.

**Problem 183:** Find the $T, N, B$ frame for $x(t) = t^2\,\mathbf{i} - \mathbf{j} + 2t\,\mathbf{k}$.