MAT 295: Exam 3 **Name:** _____
Spring – 2021
05/04/2021
80 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	20	
2	10	
3	20	
4	15	
5	20	
6	15	
Total:	100	

1. (20 points) Suppose there existed a function[1] $f(x)$ with...

$$f(x) = \frac{3x}{x+1}, \qquad f'(x) = \frac{3x(x+5)}{(x+1)(x-2)}, \qquad f''(x) = \frac{x+1}{x(x+4)}$$

(a) Find the intervals where $f(x)$ is increasing and decreasing.

(b) Find and classify the local maxima/minima values for $f(x)$.

[1]There does not exist such a function, but for sake of argument, say that there does.

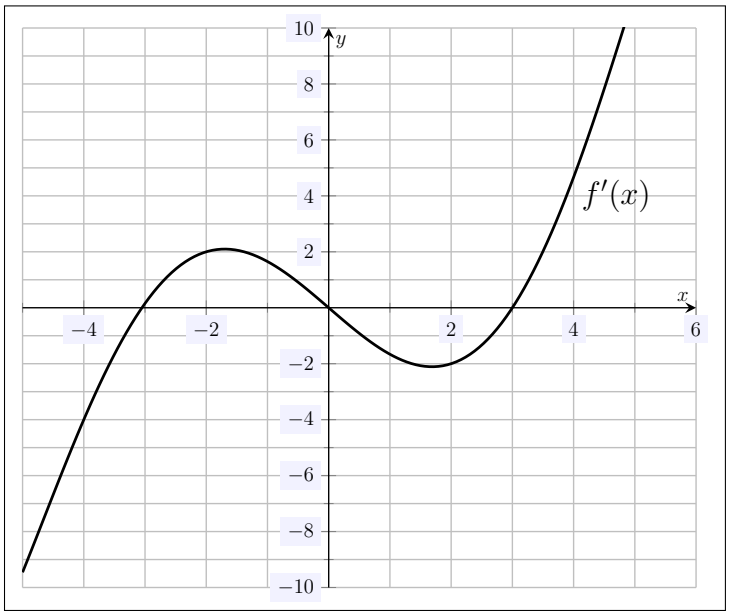
For convenience, we restate $f(x)$, $f'(x)$, and $f''(x)$ here:

$$f(x) = \frac{3x}{x+1}, \qquad f'(x) = \frac{3x(x+5)}{(x+1)(x-2)}, \qquad f''(x) = \frac{x+1}{x(x+4)}$$

(c) Find the intervals where $f(x)$ is concave up or concave down.

(d) Find any points of inflection on $f(x)$.

2. (10 points) For some function $f(x)$, a graph of $f'(x)$ is given below.



(a) Find all the intervals on which $f(x)$ is increasing or decreasing.

(b) Find all classify all the x-values of local maxima/minima for $f(x)$.

(c) Approximate all the intervals on which $f(x)$ is concave up and down.

(d) Does it appear as though $f(x)$ has a point of inflection? Explain.

3. (20 points) Let $f(x)$ be the 'parabola-like' function $f(x) = 9 - x^4$. Suppose that a rectangle is constructed so that one of its sides lies along the x-axis with two of its vertices lying on the portion of the function $f(x)$ where $f(x) \geq 0$. Of all such rectangles, what are the dimensions of the rectangle with the largest possible area? For this problem, be sure to show your dimensions give the maximal area, and state the interval on which you are optimizing.

4. (15 points) Given that the following fact from a table of integrals:

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right),$$

compute the following integral:

$$\int \frac{2^x}{2^x\sqrt{4^x+5}}\,dx$$

5. (20 points) Compute the following:

(a) $\displaystyle\int \left(\frac{1-x}{x}\right)^2 \, dx$

(b) $\displaystyle\int (\sin x - 2^x + \sec^2 x) \, dx$

(c) $\displaystyle\int_0^1 (4x - 1) \, dx$

(d) $\displaystyle\frac{d}{dx}\int_0^{2x} e^{-x^3} \, dx$

6. (15 points) Suppose that you are given the following information about a function $v(x)$:

x	0	1	2	3	4	5	6	7	8	9	10
$v(x)$	-2	8	10	0	-7	4	2	6	3	9	-3

(a) Using a right-hand sum, estimate $\int_0^{10} v(x)\,dx$ using 5 equal width rectangles.

(b) Suppose you were told that $\int_0^{10} v(x)\,dx = 13.6$ and $\int_1^{10} v(x)\,dx = 8.8$, find $\int_0^1 v(x)\,dx$.

(c) Given $p(x) = \int_0^{5x} v(2t)\,dt$, find $p'(1)$.