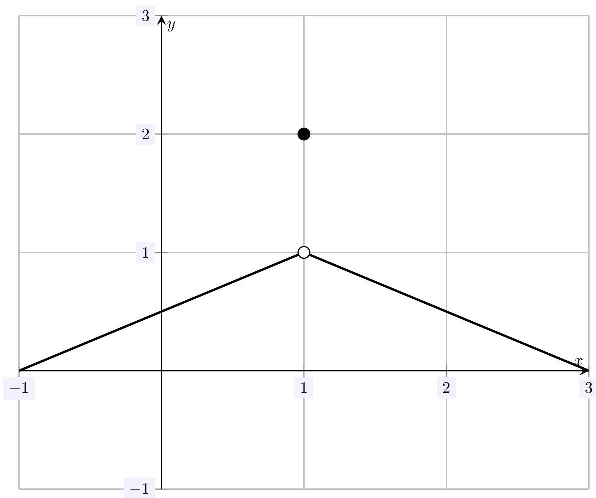
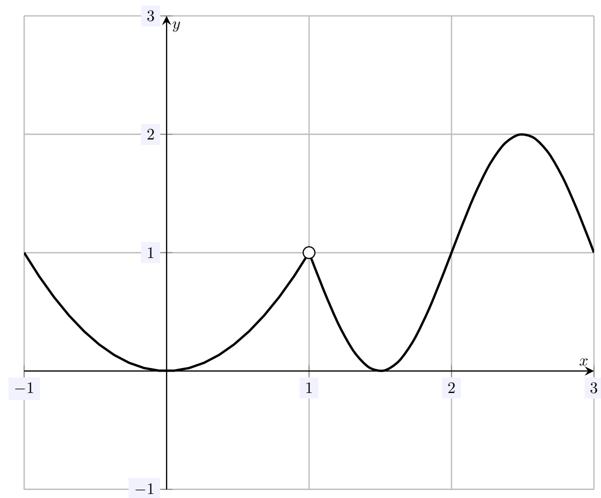
Practice Quiz 1: True/False: If $\lim_{x \to a} f(x)$ exists, then $f(a) = \lim_{x \to a} f(x)$.

Solution. The statement is *false*. If $\lim_{x \to a} f(x)$ exists, then we know that the left and right hand limit exist, i.e. $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist, and that $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$. This is a fancy way of saying that as x approaches the value a from 'any direction', the values $f(x)$ all get 'close' to the same value. But that does not mean that $f(a)$ has that same value, or that $f(x)$ is even defined at a! See the graphical examples below for an example of each, respectively.

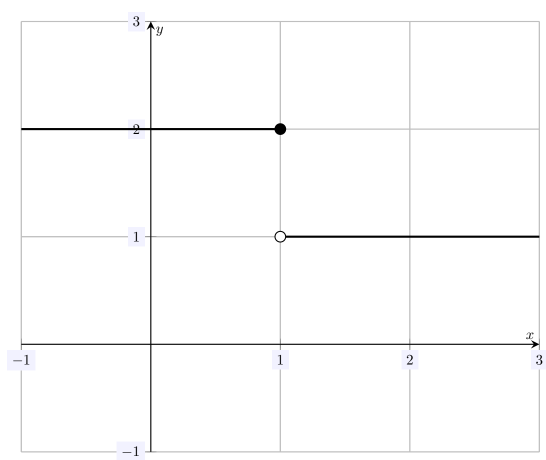




The condition that $\lim_{x \to a} f(x)$ exists and $f(a) = \lim_{x \to a} f(x)$ is the definition of $f(x)$ being *continuous* at $x = a$. □

Practice Quiz 2: True/False: If $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist, then $\lim_{x \to a} f(x)$ exists.

Solution. The statement is *false*. Remember for $\lim_{x \to a} f(x)$ to exist, both the left-hand and right-hand limits must both exist *and* be equal. Here we only know that they exist. But these two limits may be different values. Consider the example below.



For this function, $\lim_{x \to 1^-} f(x) = 2$ but $\lim_{x \to a^+} f(x) = 1$; therefore, $\lim_{x \to a} f(x)$ does not exist. □

Quiz 1: True or False: $\lim_{x\to 0} \frac{x}{\sin 3x} = 3$.

Solution. The statement is *false*. Remember the limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$. Then $\lim_{x\to 0} \frac{x}{\sin x} = 1$. Or more generally, $\lim_{\square\to 0} \frac{\square}{\sin \square} = 1$. Now both x and $3x$ tend to 0 as $x \to 0$. We just need to make them match. We have

$$\lim_{x\to 0} \frac{x}{\sin 3x} = \lim_{x\to 0} \frac{x}{\sin 3x} \cdot \frac{3}{3} = \lim_{x\to 0} \frac{3x}{\sin 3x} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

Therefore, the statement is false. □

Quiz 2: True or False: $\lim_{x\to\infty} \frac{x + e^{-x}}{x^2 + \ln x} = \infty$.

Solution. The statement is *false*. We use growth rates to compute the limit. The dominating term in the numerator is x. [Note that it is not e^{-x} because this does not grow as $x \to \infty$. In fact, $\lim_{x\to\infty} e^{-x} = 0$.] The dominating term in the denominator is x^2. Therefore for 'large' x, we have $\frac{x + e^{-x}}{x^2 + \ln x} \approx \frac{x}{x^2} = \frac{1}{x} \to 0$ as $x \to \infty$. So, we believe that

$$\lim_{x\to\infty} \frac{x + e^{-x}}{x^2 + \ln x} = 0$$

Therefore, the statement is false. □

Quiz 3: True/False: $\lim_{x\to 0} \frac{x^2}{|x|} = 0$.

Solution. The statement is *true*. Remember with absolute value type limits, we often want to examine the left and right hand limits.

$$\text{Left-Hand Limit: } \lim_{x\to 0^-} \frac{x^2}{|x|} = \lim_{x\to 0^-} \frac{x^2}{-x} = \lim_{x\to 0^-} -x = 0$$

$$\text{Right-Hand Limit: } \lim_{x\to 0^+} \frac{x^2}{|x|} = \lim_{x\to 0^+} \frac{x^2}{x} = \lim_{x\to 0^+} x = 0$$

Because the right and left hand limits exist and are equal, we know that

$$\lim_{x\to 0} \frac{x^2}{|x|} = 0$$

Therefore, the statement is true.

Quiz 4: True/False: $\lim_{x\to\infty} \dfrac{3x^2+2x-1}{x-2x^2} = -\dfrac{3}{2}$.

Solution. The statement is *true*. Remember that for rational functions,

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \begin{cases} \pm\infty, & \deg f > \deg g \\ 0, & \deg f < \deg g \\ \text{Ratio Coefficients}, & \deg f = \deg g \end{cases}$$

The degree of the numerator and denominator is 2. Therefore, the limit is the ratio of the coefficients, which are 3 and -2, respectively. If we had to show this limit 'rigorously', remember that we multiple by 1 over the dominating term in the denominator, all over itself.

$$\begin{aligned} \lim_{x\to\infty} \frac{3x^2+2x-1}{x-2x^2} &= \lim_{x\to\infty} \frac{3x^2+2x-1}{x-2x^2} \cdot \frac{1/x^2}{1/x^2} \\ &= \lim_{x\to\infty} \frac{\dfrac{3x^2}{x^2} + \dfrac{2x}{x^2} - \dfrac{1}{x^2}}{\dfrac{x}{x^2} - \dfrac{2x^2}{x^2}} \\ &= \lim_{x\to\infty} \frac{3+2/x-1/x^2}{1/x-2} \\ &= \frac{3+0-0}{0-2} \\ &= -\frac{3}{2} \end{aligned}$$

Quiz 5: True/False: If $f(1)$ and $f'(1) = -2$, then the tangent line to $f(x)$ at $x = 1$ is $y = 3 - 2x$.

Solution. The statement is *false*. We know that the tangent line to a function $f(x)$ at $x = a$ is $y = f(a) + f'(a)(x-a)$. Then we have

$$y = 5 + (-2)(x-1) = 5 - 2x + 2 = 7 - 2x$$

Therefore, the tangent line should be $y = 7 - 2x$. The given 'tangent line' results from the mistake of distributing the -2 to the -1, which obtains -2 rather than the correct $+2$. Be careful with negatives!

Quiz 6: True/False: If a function is differentiable, then it is continuous.

Solution. The statement is *true*. Remember, if a function is differentiable at a point, then it must be continuous at that point. However, the reverse direction is false. For example, the function $f(x) = |x|$ is continuous everywhere but not differentiable at $x = 0$. Remember, this can even be taken to the extreme as there are functions which are continuous everywhere but differentiable nowhere!

Quiz 7: True/False: The function $f(x) = \sqrt{x}$ is differentiable at $x = 0$.

Solution. The statement is *false*. An intuitive (but not quite correct answer) is that if we apply the derivative rules, we obtain $f'(x) = \dfrac{1}{2\sqrt{x}}$, which is not defined at $x = 0$. This type of approach will often work—but this does not work for all functions. If you encounter a problem like this, it is best to check it from the definition of the derivative. We have

$$\begin{aligned} f'(0) &:= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h\to 0} \frac{f(h) - f(0)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{h} - 0}{h} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{h}} \\ &= \infty \end{aligned}$$

Therefore, $f(x) = \sqrt{x}$ is not differentiable at $x = 0$.

Quiz 8: True/False: $\dfrac{d}{dx}\big(\ln(x)\arcsin(x^2)\big) = \dfrac{1}{x} \cdot \arcsin(x^2) + \ln x \cdot \left(\dfrac{1}{\sqrt{1-x^2}} \cdot 2x\right)$

Solution. The statement is *false*. Remember to be careful with certain types of derivatives—especially arc-trig derivatives. Using the box approach, we have

$$\frac{d}{dx} \arcsin \Box = \frac{1}{\sqrt{1 - \Box^2}} \cdot \text{Chain Rule “Stuff”}$$

Here $\Box$ is x^2. So we have

$$\frac{d}{dx} \arcsin(x^2) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

This means that the correct derivative is. . .

$$\frac{d}{dx}\big(\ln(x)\arcsin(x^2)\big) = \frac{1}{x} \cdot \arcsin(x^2) + \ln x \cdot \left(\frac{1}{\sqrt{1-x^4}} \cdot 2x\right)$$

Quiz 9: True/False: $\dfrac{d}{dx}(x^2 + e^y - xy^2) = 2x + e^y y' - (y^2 + 2xyy')$

Solution. The statement is *true*. This is a simple application of implicit differentiation. Note that $\frac{d}{dx} e^y = e^y \cdot y'$, and to implicitly differentiate xy^2, we need the product rule. We then have

$$\frac{d}{dx}(x^2 + e^y - xy^2) = 2x + e^y y' - (y^2 + 2xyy')$$

Quiz 10: True/False: The tangent line to $\frac{x}{y} = x^2 - 2y^4$ at the point $(2, 1)$ is $y = 1 + \frac{5}{8}(x - 2)$.

Solution. The statement is *false*. Note that we could differentiate things "as they sit." However, it will be easier to multiply both sides by y first to avoid a quotient rule. We have

$$\begin{aligned} \frac{x}{y} &= x^2 - 2y^4 \\ x &= x^2 y - 2y^5 \\ \frac{d}{dx}\, x &= \frac{d}{dx}\left(x^2 y - 2y^5\right) \\ 1 &= (2xy + x^2 y') - 10y^4 y' \end{aligned}$$

From this, we evaluate at $(2, 1)$ to find $\frac{dy}{dx}$.

$$\begin{aligned} \Big[1 =& 2xy + x^2 y' - 10y^4 y'\Big]_{(2,1)} \\ 1 &= 4 + 4y' - 10y' \\ -3 &= -6y' \\ \frac{dy}{dx} &= \frac{1}{2} \end{aligned}$$

Then the tangent line is $y = 1 + \frac{1}{2}(x - 2)$.

Quiz 11: True/False: If $y^2 = x^3 + x$, then $\frac{d^2y}{dx^2} = \frac{(2y)(6x) - \left(2 \cdot \frac{3x^2+1}{2y}\right)(3x^2+1)}{(2y)^2}$.

Solution. The statement is *true*. First, we find $\frac{dy}{dx}$.

$$\begin{aligned} \frac{d}{dx}\, y^2 &= \frac{d}{dx}\left(x^3 + x\right) \\ 2yy' &= 3x^2 + 1 \\ \frac{dy}{dx} &= \frac{3x^2+1}{2y} \end{aligned}$$

Then we can find $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{3x^2+1}{2y}\right) = \frac{(2y)(6x) - (2y')(3x^2+1)}{(2y)^2} = \frac{(2y)(6x) - \left(2 \cdot \frac{3x^2+1}{2y}\right)(3x^2+1)}{(2y)^2}$$

Quiz 12: True/False: $\dfrac{d}{dx}\, x \arctan(\log_2(x)) = \arctan(\log_2(x)) + x \cdot \dfrac{1}{1+x^2} \cdot \dfrac{1}{x}$

Solution. The statement is *false*. We simply apply the product and chain rules:

$$\frac{d}{dx}\, x \arctan(\log_2(x)) = 1 \cdot \arctan(\log_2(x)) + x \cdot \frac{1}{1+(\log_2 x)^2} \cdot \frac{1}{x \ln 2}$$

The error here comes from the common mistake that $\frac{d}{dx} \arctan \Box = \frac{1}{1+\Box^2}$ and then take into account the chain rule. For example, $\frac{d}{dx} \arctan(\sin x) = \frac{1}{1+(\sin x)^2} \cdot \cos x$.

Quiz 13: True/False: $\dfrac{d}{dx}\, 2^x \log_\pi(x^2+1) \csc(2x) = 2^x \ln 2 \cdot \log_\pi(x^2+1) \cdot \csc(2x) + 2^x \cdot \dfrac{2x}{(x^2+1)\ln \pi} \cdot \csc(2x) + 2^x \cdot \log_\pi(x^2+1) \cdot (-\csc x \cot x \cdot 2)$

Solution. The statement is *false*. We simply apply the product and chain rule:

$$\begin{aligned}\text{True/False: } \frac{d}{dx}\, 2^x \log_\pi(x^2+1) \csc(2x) = 2^x \ln 2 \cdot \log_\pi(x^2+1) \cdot \csc(2x) \\ + 2^x \cdot \frac{2x}{(x^2+1)\ln \pi} \cdot \csc(2x) \\ + 2^x \cdot \log_\pi(x^2+1) \cdot (-\csc(2x)\cot(2x) \cdot 2)\end{aligned}$$

The error here comes from the common mistake that $\frac{d}{dx} \csc \Box = -\csc \Box \cot \Box$ and then take into account the chain rule. For example, $\frac{d}{dx} \csc(e^x) = -\csc(e^x)\cot(e^x) \cdot e^x$.

Quiz 14: True/False: $\dfrac{d}{dx} \dfrac{\cos^2(2x)}{3^x} = \dfrac{3^x \cdot (-\sin^2(2x) \cdot 2) - (3^x \ln 3) \cdot (1-2x)^2}{9^x}$

Solution. The statement is *false*. Simply apply the quotient and chain rule:

$$\begin{aligned}\frac{d}{dx} \frac{\cos^2(2x)}{3^x} &= \frac{3^x \cdot (2\cos(2x) \cdot \sin(2x) \cdot 2) - (3^x \ln 3) \cdot (1-2x)^2}{(3^x)^2} \\ &= \frac{3^x \cdot (4 \sin(2x)\cos(2x)) - (3^x \ln 3) \cdot (1-2x)^2}{9^x}\end{aligned}$$

Note that $(3^x)^2 = 3^{2x} = (3^2)^x = 9^x$. The error here comes from the common error when differentiating things 'like' $\cos^2 x$. Remember, the power rule applies first. For example, $\frac{d}{dx} \sin^2(e^x) = 2\sin(e^x) \cdot \cos(e^x) \cdot e^x$.

Quiz 15: True/False: If $f(x) = \sin^2(\pi x)$, then the linearization of $f(x)$ at $x = \frac{1}{6}$ is $\ell(x) = \dfrac{1}{4} + \dfrac{\pi\sqrt{3}}{2}\left(x - \dfrac{1}{6}\right)$.

Solution. The statement is *true*. The linearization of a function at $x = a$ is just the tangent line at $x = a$. We have

$$f\left(\frac{1}{6}\right) = \sin^2\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$
$$f'(x) = 2\sin(\pi x)\cdot\cos(\pi x)\cdot\pi\Big|_{x=\frac{1}{6}} = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\pi = \frac{\pi\sqrt{3}}{2}$$

Then the linearization, $\ell(x)$, for $f(x)$ at $x = \frac{1}{6}$ is

$$\ell(x) = \frac{1}{4} + \frac{\pi\sqrt{3}}{2}\left(x - \frac{1}{6}\right)$$

Quiz 16: True/False: $\lim\limits_{x\to 0} \dfrac{x^2}{1-\cos x} = 2$

Solution. The statement is *true*. Using l'Hôpital's twice (because we get a $0/0$ form), we obtain...

$$\begin{aligned}
\lim_{x\to 0}\frac{x^2}{1-\cos x} &\overset{\text{LH}}{=} \lim_{x\to 0}\frac{2x}{\sin x}\\
&\overset{\text{LH}}{=} \lim_{x\to 0}\frac{2}{\cos x}\\
&= \frac{2}{\cos 0}\\
&= \frac{2}{1}\\
&= 2
\end{aligned}$$

Quiz 17: True/False: $\lim_{x\to\infty}\dfrac{2x+\sin x}{1-3x} \overset{\text{LH}}{=} \lim_{x\to\infty}\dfrac{1+\sin x}{-3} = \text{DNE}$

Solution. The statement is *false*. Remember, l'Hôpital's Rule only says that $\lim f/g = \lim f'/g'$ *if* the limit $\lim f'/g'$ exists. If $\lim f'/g'$ does not exist, we cannot use this (via l'Hôpital's) to determine the limit of $\lim f/g$. Maybe it exists; maybe it doesn't. But l'Hôpital's Rule says nothing. Here the 'trick' is an old one:

$$\begin{aligned}
\lim_{x\to\infty}\frac{2x+\sin x}{1-3x} &= \lim_{x\to\infty}\frac{2x+\sin x}{1-3x}\cdot\frac{1/x}{1/x}\\
&= \lim_{x\to\infty}\frac{2+\frac{\sin x}{x}}{1/x-3}\\
&= \frac{2+0}{0-3}\\
&= -\frac{2}{3}
\end{aligned}$$

Quiz 18: True/False: If $f(x) = x^2 + 2\sin x$, then there is a value $r \in [0, 3]$ so that $f(r) = 2.5$.

Solution. The statement is *true*. Notice that x^2 is continuous (it is a polynomial) and $2\sin x$ is continuous ($\sin x$ is continuous). Then $f(x) = x^2 + 2\sin x$ is continuous because it is a sum of continuous functions. Now

$$\begin{aligned} f(0) &= 0 + 2(0) = 0 \\ f(3) &= 3^2 + 2\sin 3 = 9 + 2\sin 3 \geq 9 - 2 = 7 \end{aligned}$$

I have no idea what $2\sin 3$ is (without a calculator). But I know $-1 \leq \sin x \leq 1$ so that $-2 \leq 2\sin x \leq 2$. Pretending $2\sin 3$ is as small as possible, I know that $f(3) = 9 + 2\sin 3 \geq 9 - 2 = 7$, as stated above. Now $f(0) = 0 \leq 2.5 \leq 7 \leq f(3)$. But because $f(x)$ is continuous, the Intermediate Value Theorem (IVT) says there is a value $c \in [0, 3]$ such that $f(r) = 2.5$.

Quiz 19: True/False: The value c guaranteed by the MVT for $f(x) = 2x^3 + 6x + 1$ on the interval $[0, 1]$ is $c = 1/\sqrt{3}$.

Solution. We know that $f(x)$ is continuous on $[0, 1]$ (it is continuous everywhere because it is a polynomial). We know that $f(x)$ is differentiable on $(0, 1)$ because $f'(x) = 6x^2 + 6$ is defined on $(0, 1)$. Then by the MVT, there is a $c \in (0, 1)$ so that

$$\begin{aligned} f'(c) &= \frac{f(1) - f(0)}{1 - 0} \\ 6c^2 + 6 &= \frac{9 - 1}{1} \\ 6c^2 + 6 &= 8 \\ 6c^2 &= 2 \\ c^2 &= \frac{1}{3} \\ c &= \pm\frac{1}{\sqrt{3}} \end{aligned}$$

But $-1/\sqrt{3} \notin (0, 1)$. Therefore, it must be that $c = 1/\sqrt{3}$.

Quiz 20: True/False: The equation $x^7 + x^5 = 1 - x^3$ has exactly one solution.

Solution. The statement is *true*. First, move things over to one side: $x^7 + x^5 + x^3 - 1 = 0$. Then the equation has exactly one solution if the function $f(x) = x^7 + x^5 + x^3 - 1$ has exactly one root. Now $f(x) = x^7 + x^5 + x^3 - 1$ is continuous because it is a polynomial. Observe that $f(0) = -1$ and $f(1) = 2$. By the Intermediate Value Theorem, there is a $c \in [0, 1]$ such that $f(c) = 0$. But then $c^7 + c^5 + c^3 - 1 = 0$ which means that $c^7 + c^5 = 1 - c^3$, so that the equation has at least one solution. Suppose there were two solutions (or more, so take any two of them): s_1 and s_2. Then $f(s_1) = 0 = f(s_2)$. We know $f'(x) = 7x^6 + 5x^4 + 3x^2$ (a polynomial so it is defined everywhere) so that $f(x)$ is differentiable on $(0,1)$. By the Mean Value Theorem (MVT) applied to the interval with s_1, s_2 as endpoints, we have a c in that interval so that

$$\begin{aligned} f'(c) &= \frac{f(s_2) - f(s_1)}{s_2 - s_1} \\ 7c^6 + 5c^4 + 3c^2 &= \frac{0 - 0}{s_2 - s_1} \\ 7c^6 + 5c^4 + 3c^2 &= 0 \end{aligned}$$

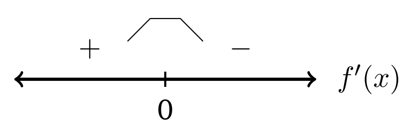
But the left side is positive unless $c = 0$. But $c \in (0,1)$ so that c cannot be 0. This must mean there is actually is not two or more solutions. We used the IVT to show there was one solution. So there must be exactly one solution.

Quiz 21: True/False: If $f'(2) = 0$ and $f''(2) > 0$, then $(2, f(2))$ is a local max.

Solution. The statement is *true*. This is just the second derivative test applied to the point $(2, f(2))$ for the function $f(x)$.

Quiz 22: True/False: If $f(x) = 1/x^2$, because $f'(x) > 0$ for $x < 0$ and $f'(x) < 0$ for $x > 0$, $x = 0$ is a local max.

Solution. The statement is *false*. Applying the first derivative test naïvely to the function $f(x)$,



so that it would appear that $x = 0$ is location of a minimum value (point) for $f(x)$. However, the function $f(x)$ needs to be defined at $x = 0$ for this to correspond to a maximum or minimum point/value. Observe that $f(x) = 1/x^2$ is not defined at $x = 0$, so that $x = 0$ is the location of neither a maximum or minimum.

Quiz 23: True/False: The only critical value for $f(x) = x + 3\log(x - 2)$ is $x = -1$.

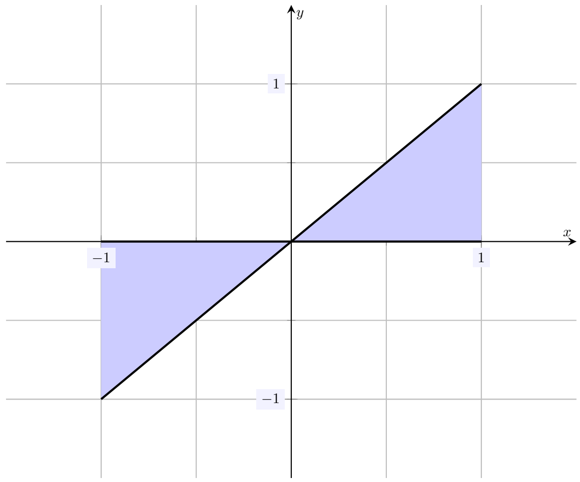
Solution. The statement is *false*. To find critical values for a function, we look for x-values where $f'(x) = 0$ or where $f'(x)$ is undefined. We have

$$f'(x) = 1 + \frac{3}{x-2} = \frac{x-2}{x-2} + \frac{3}{x-2} = \frac{x+1}{x-2}$$

This is undefined at $x = 2$. Setting this to 0, we see that $x + 1 = 0$ so that $x = -1$. We then have two critical values: $x = -1$ and $x = 2$.

Quiz 24: True/False: $\int_{-1}^{1} x\ dx = 1$

Solution. The statement is *false*. Recall that area under the x-axis is counted as negative.



The area of each of these triangles is equal to $\frac{1}{2}bh = \frac{1}{2} \cdot 1 \cdot 1 = 1/2$. Then we have

$$\int_{-1}^{1} x\ dx = \frac{1}{2} - \frac{1}{2} = 0.$$

Quiz 25: True/False: $\int \cos x\ dx = \sin x$

Solution. The statement is *false*. We can only ever know an indefinite integral up to a constant, C, because $\frac{d}{dx}\big(f(x) + C\big) = f'(x)$. Therefore, we have

$$\int \cos x\ dx = \sin x + C.$$

Quiz 26: True/False: $\int xe^x\ dx = xe^x - e^x + C$

Solution. The statement is *true*. Although, we do not have the integration techniques at hand to come up with an antiderivative for $\int xe^x\ dx$ on our own, given a possible antiderivative, we can simply check if it is a solution by differentiating to see if we obtain the integrand:

$$\frac{d}{dx}\ (xe^x - e^x + C) = (e^x + xe^x) - e^x + 0 = xe^x$$

Quiz 27: True/False: $\int_1^{e^2} \frac{1}{x}\, dx = 2$

Solution. The statement is *true*. This is a matter of routine integration:

$$\int_1^{e^2} \frac{1}{x}\, dx = \ln|x|\Big|_1^{e_2} = \ln e^2 - \ln 1 = 2 - 0 = 2$$

Quiz 28: True/False: $\int \sin\left(\frac{x}{\pi}\right)\, dx = \pi \cos x + C$

Solution. The statement is *false*. Though we have not formally learned u-substitution, again, we can always check a given antiderivative by differentiating:

$$\frac{d}{dx}\left(\pi \cos x + C\right) = -\pi \sin x$$

The integrand is off by a sign, but also the argument must be divided by π—and the given one is not; hence, the answer is false. If we had to "guess-and-check", we know that the derivative of $\cos x$ is $-\sin x$, so we might want to guess $-\cos x$. To have an argument of $\sin(x/\pi)$, the argument of our guess must be the same. So we may guess $-\cos(x/\pi) + C$.

$$\frac{d}{dx}\left(-\cos\left(\frac{x}{\pi}\right) + C\right) = \sin\left(\frac{x}{\pi}\right) \cdot \frac{1}{\pi}$$

Then we need only "tweek" our guess by a factor of π. So we should have an answer of $-\pi\cos\left(\frac{x}{\pi}\right) + C$, as we can check:

$$\frac{d}{dx}\left(-\pi\cos\left(\frac{x}{\pi}\right) + C\right) = \pi \sin\left(\frac{x}{\pi}\right) \cdot \frac{1}{\pi} = \sin\left(\frac{x}{\pi}\right)$$

Quiz 29: True/False: $\int e^{\cos x} \sin x\, dx = e^{\cos x} + C$

Solution. The statement is *false*. Now that we know the formal process of u-substitution, we can avoid our 'guess-and-check' method—though it makes for good practice and understanding of what u-substitution is doing. Let $u = \cos x$. Then we have $du = -\sin x\, dx$ so that $dx = \dfrac{du}{-\sin x}$. Then

$$\int e^{\cos x} \sin x\, dx = \int e^u \sin x\, \frac{du}{-\sin x} = -\int e^u\, du = -e^u + C = -e^{\cos x} + C$$

We can also let $u = \cos x$, so that $du = -\sin x\, dx$, and then 'prep' our integral before substitution:

$$\int e^{\cos x} \sin x\, dx = -\int e^{\cos x} \cdot -\sin x\, dx = -\int e^u\, du = -e^u + C = -e^{\cos x} + C$$

Quiz 30: True/False: $\int 8^{1-x}\,dx = -8^{1-x} + C$

Solution. The statement is *false*. This is a u-substitution. Let $u = 1 - x$. Then $du = -\,dx$. Now

$$\int 8^{1-x}\,dx = -\int 8^u\,du = -\frac{8^u}{\ln 8} + C = -\frac{8^{1-x}}{\ln 8} + C$$

Quiz 31: True/False: $\int \frac{1}{\sqrt{2x - x^2}}\,dx = C - \arcsin(1 - x)$

Solution. The statement is *true*. The integrand *vaguely* looks like the derivative of $\arcsin x$. We will make this "look" to be the case. First, we complete the square for the root in the denominator.

$$\begin{aligned}
\int \frac{1}{\sqrt{2x - x^2}}\,dx &= \int \frac{1}{\sqrt{1 - 1 + 2x - x^2}}\,dx \\
&= \int \frac{1}{\sqrt{1 - (1 - 2x + x^2)}}\,dx \\
&= \int \frac{1}{\sqrt{1 - (x^2 - 2x + 1)}}\,dx \\
&= \int \frac{1}{\sqrt{1 - (x - 1)^2}}\,dx
\end{aligned}$$

Now let $u = x - 1$, then $du = \,dx$. We then have

$$\int \frac{1}{\sqrt{1 - (x-1)^2}}\,dx = \int \frac{1}{\sqrt{1 - u^2}}\,du = \arcsin u + C = \arcsin(x - 1) + C$$

This seems different than the given answer. However, we can also check:

$$\frac{d}{dx}\left(C - \arcsin(1 - x)\right) = -\frac{1}{\sqrt{1 - (1 - x)^2}} \cdot -1 = \frac{1}{\sqrt{1 - (1 - 2x + x^2)}} = \frac{1}{\sqrt{2x - x^2}}$$

So the given answer is an antiderivative. Then our answer and the given answer must be the same up to a constant. In fact, they are exactly equal because $\arcsin x$ is an odd function, i.e. $\arcsin(-x) = -\arcsin x$. With this fact, we have

$$\arcsin(x - 1) + C = \arcsin(-(1 - x)) + C = -\arcsin(1 - x) + C = C - \arcsin(1 - x)$$