Challenge Integrals

Challenge Rules: The following are a series of more challenging integrals and series, all of which are solvable using techniques that you will learn this semester. While all of the integrals are difficult, some are more difficult than others—appearances can be deceiving!

Solutions must be written up in full and given to me. Using WolframAlpha or any other source other than the hints given here is an automatic disqualification! All integrals will come on a first come, first serve basis—meaning once one person has solved it, you are not able to receive any 'points' for solving it. The student(s) with the most points at the end of the semester will receive some sort of prize. Perhaps some prizes will be given to other people that solve some of these—we shall see.

Good luck and may the odds be ever in your favor!

Problem 1: $\displaystyle\int_2^4 \frac{\sqrt{\ln(9-(6-x))}}{\sqrt{\ln(9-x)}+\sqrt{\ln(3-x)}}\,dx$

Hint: Use the following: $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Problem 2: $\displaystyle\int_0^\infty \frac{\sqrt[3]{x}}{1+x^2}\,dx$

Problem 3: $\displaystyle\int_0^1 \frac{\ln(x+1)}{x^2+1}\,dx$

Hint: You might want to use the following identity: $\tan(x\pm y)=\dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Problem 4: $\displaystyle\int e^x\sqrt{1+e^{2x}}\,dx$

Problem 5: $\displaystyle\int \frac{1-x^2}{1+3x^2+x^4}\,dx$

Problem 6: For $n=2,3,4,\ldots$, find a formula for $\int \frac{dx}{x^n-x}$

Problem 7: $\displaystyle\int \frac{dx}{(x^2+1)\sqrt{x^2-1}}$

Problem 8: $\displaystyle\int \frac{dx}{3e^x+1}$

Problem 9: $\displaystyle\int_0^{\pi/2} \frac{\ln(\sin x)\ln(\cos x)}{\tan x}\,dx$

Hint: You will want to use the Riemann Zeta function: $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$

Problem 10: $\displaystyle\int_0^{\pi/4} x\left(\frac{(1-x^2)\ln(1+x^2)+(1+x^2)-(1-x^2)\ln(1-x^2)}{(1-x^4)(1+x^2)}\right)e^{\frac{x^2-1}{x^2+1}}\,dx$

Problem 11: $\displaystyle\int_0^{1} \frac{1-x}{(1+x)\ln x}\,dx$

Hint: You will want to make use of the properties of $\log$.

Problem 12: $\displaystyle\int_0^{1} \frac{\ln^2(1-x)}{x}\,dx$

Hint: You will want to use the Riemann Zeta function: $\displaystyle\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$

Problem 13: $\displaystyle\int_0^{1} \left(\frac{\ln x}{1-x}\right)^2 dx$

Hint: You will want to make extensive use of a particular value of the Riemann Zeta function: $\displaystyle\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

Problem 14: $\displaystyle\int_0^{\infty} \frac{dx}{(x+\sqrt{1+x^2})^n}$

Problem 15: $\displaystyle\int_0^{1} \ln\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Problem 16: $\displaystyle\left(\int_{-\pi/2}^{0} \sin(2016x)\sin^{2014} x\,dx\right)^{-1}$

Problem 17: $\displaystyle 4030\sum_{x=0}^{\infty} \frac{x}{2^{x+2}}$