

# Mathematical Proficiency Exam

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(1) Find  $\theta$ . [Hint: Look up]

(2) Recall Fermat's theorem of the sum of squares: If  $p \equiv 1(4)$  is prime, then  $p$  is the sum of squares. (a) Write 8 as the sum of the squares  $\square$  and  $\square$ , (b) why does this prove that 8 is prime?

(3) Find three people and tell them about quadratic reciprocity. Why does nobody care? [Hint: Consider a simple case first, like a family member]

(4) Let  $K$  be a field, and  $V$  a  $K$ -vector space. If  $G$  is a group, not necessarily abelian, and  $\rho : G \rightarrow GL(V)$  an irreducible  $G$ -representation, let  $k = \dim V$ . What is  $\chi(\rho)$ ? [Hint: It is not  $k$ ]

(5) Consider [twitter.com/Pontifex](https://twitter.com/Pontifex). What does this imply about the existence of strong limit cardinals? In the universe of such a cardinal, can club sets exist?

(6) State and prove three properties of  $SU(3) \times \mathbb{R}P^6$  in the context of the contemporary conversation on gun control. Can increased firearm regulation work in  $SU(3) \times \mathbb{R}P^6$ , and if not, who is to blame?

(7) Construct a joke involving Radon measures and probiotic yogurt.

(8) The snake from the Snake Lemma is trying to kill the butterfly from the Butterfly Lemma. What can we tensor them over to preserve the butterfly?

